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Analysis of an Incremental Digital Positioning Servosystem with Digital Rate Feedback

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by

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ABSTRACT

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A technique is presented which may be used to measure the average rate of a series of pulses in a continuous manner. The value of this technique is demonstrated by utilizing it in an incremental digital positioning servosystem to provide first-derivative feedback. An experimental model of such a system is studied with respect to its response to a step input for various amounts of digital rate feedback. The dynamic performance is compared with the response predicted by a mathematical model of the system. It is further compared with the response of the same system in which an analog tachometer is used.

The high degree of correlation of the separate responses indicates that the rate measurement technique is practically applicable to digital compensation. Author

TABLE OF CONTENTS

Page

LIST OF FIGURES

Figure	Title	Page
18	Typical Segment of Quantizer Disc	44
19	Typical Shaped Output of Schmitt Triggers with Quantizer Rotating at Constant Speed in Forward Direction	44
20	Wang Laboratories No. 43 Encoder Circuit Schematic	45
21	Wang Laboratories No. 216 Schmitt Triggers	47
22	Quantizer Direction Sensing Logic	48
23	Wang Laboratories No. 242 Direction Sensing Schematic	49
24	Synchronization and Timing Logic	52
25	4-Bit Bidirectional Magnitude and Sign Binary Counter	54
26	4-Bit Sign and Magnitude Bidirectional Counter	58
27	Shift Register	60
28	Digital to Analog Converter Voltage Ladder	62
29	Determination of Output Voltage	62
30	Wang Laboratories No. 215 Digital to Analog Converter	64
31	Phase Controlled Modulator	66
32	Servo Amplifier	68
33	Computer Program	70
34	Computer Flow Diagram	72
35	Time Determination of Quanta Boundary Crossing	74

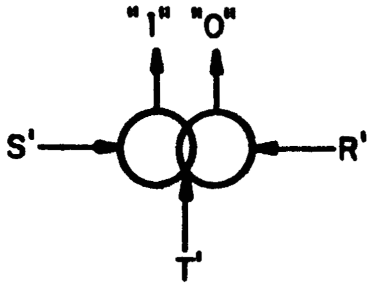
LIST OF SYMBOLS

A_n	magnitude and sign of n^{th} reference pulse
B	backward rotation of quantizer
BKWD	decrease magnitude of counter
B_m	magnitude and sign of m^{th} feedback pulse
D	a binary number
$E_{o,n}$	output voltage of D/A converter due to n^{th} bit
e	natural logarithm base 2.718
e_n	voltage of n^{th} bit in D/A converter
$e_1(t)$	value of POSITION bidirectional counter
$e_2(t)$	value of RATE bidirectional counter
e(t)	sum of position and rate voltages
F	forward rotation of quantizer
FWD	increase magnitude of counter
f_i	maximum input pulse frequency
f_{sh}	shift frequency
f_{sync}	synchronization frequency
f(t)	value of feedback pulses from quantizer
G(s)	gain of continuous portion of system
K	continuous system gain
K_p	position loop gain

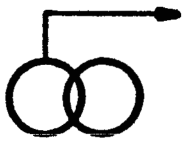
K_T	tachometer loop gain
k	an integer
$\mathcal{L}$	Laplace transform operation
$\mathcal{L}^{-1}$	inverse Laplace transformation
m	an integer
N_{sh}	number of pulses in shift register
N	negative count input pulse
N'	negative sign in counter
n	an integer
n_{sh}	number of memory elements in shift register
P	positive count input pulse
P'	positive sign in counter
P_{sh}	shift pulse
P_t	pulse occurring at time t
q(t)	response of continuous portion of system to a step input
R	value of resistors in D/A converter
R_{av}	average input pulse rate
R_T	value of rate bidirectional counter at time T
R'_n	reset input to flip-flop element
S'_n	set input to flip-flop element
s	Laplace operator
T	time parameter
T'_n	trigger input to flip-flop element

T_L	time constant due to motor and gear train inertia
T_M	time constant due to electrical characteristics of the motor
T_{fm}	time at which m^{th} feedback pulse occurs
T_{rn}	time at which n^{th} reference pulse occurs
t	time parameter
u(t)	unit input at time t
ZERO	magnitude of counter is zero
α	logic transition $0 \rightarrow 1$
β	logic transition $1 \rightarrow 0$
$\delta(t)$	impulse at time t
ζ	damping coefficient
$\theta_i(t)$	reference input
$\theta_o(t)$	relative output position (quanta)
τ	shift register delay time
ω_n	natural frequency

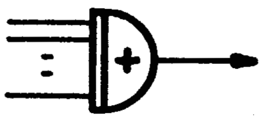
LOGICAL SYMBOLS



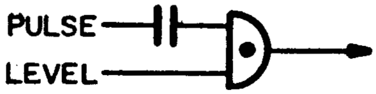
FLIP - FLOP



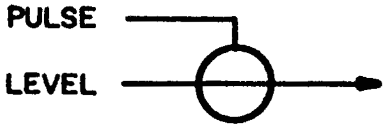
FREE - RUNNING MULTIVIBRATOR



NOR GATE



GATED PULSE GENERATOR



STEERING GATE

CHAPTER I
NUMERICAL CONTROL TECHNIQUES

Introduction

The rapid increase in the use of numerical control systems has stimulated interest in the analysis and design of various types of digital control loops.

This report is concerned with a technique which has been developed to provide a continuous measurement of the average rate of pulses entering the device. Although there are many applications for such a unit, one which is very important in the numerical control field is to provide digital compensation to improve the response of a closed loop digital system.

Incremental System

Two basic types of digital feedback systems are in current use. The first, as shown in Figure 1, is an incremental system. A quantizer is used to measure the output of the controlled variable. A pulse is emitted on one of two lines whenever a "quanta" boundary is crossed indicating both the transition and the corresponding direction of rotation. The feedback pulses enter a bidirectional counter which indicates the difference in quanta between the actual position and the desired position. The disadvantage of such a system is that any pulses lost or gained in the input lines to the bidirectional counter or in the quantizer unit will produce a permanent offset of the output.

Absolute System

The second type of feedback control system, shown in Figure 2, is an "absolute" system. In this unit, an absolute encoder is used to determine the position of the output. This position is then compared with a reference input which consists of the desired position coded in an absolute form. The difference is used to actuate the output until the desired position is reached. The disadvantage of this system is that the absolute encoder, its associated read logic, and the comparison logic are all considerably more complex than the units used in the incremental system. As a result, there is a considerable difference in cost between the two systems.

Hybrid System

It is also possible to combine the above two methods. The result is a system which is more reliable than the first but less expensive than the second. This is achieved by designing an incremental quantizer with a limited number of fixed positions coded in an absolute code. Whenever one of the fixed positions is crossed, a bidirectional counter, used to indicate the actual position of the output, is immediately corrected to the proper value. Of course, it is necessary to use a comparison unit and an absolute reference; however, the cost of the encoder and readout logic is reduced considerably.

The economic advantage of the incremental system and the variations which might improve its reliability indicate that a continuation in the investigation of incremental control systems is of considerable value.

In the following sections of this report, the rate measurement technique will be described and its usefulness will be analyzed by using it to provide rate feedback in an incremental digital positioning servosystem.

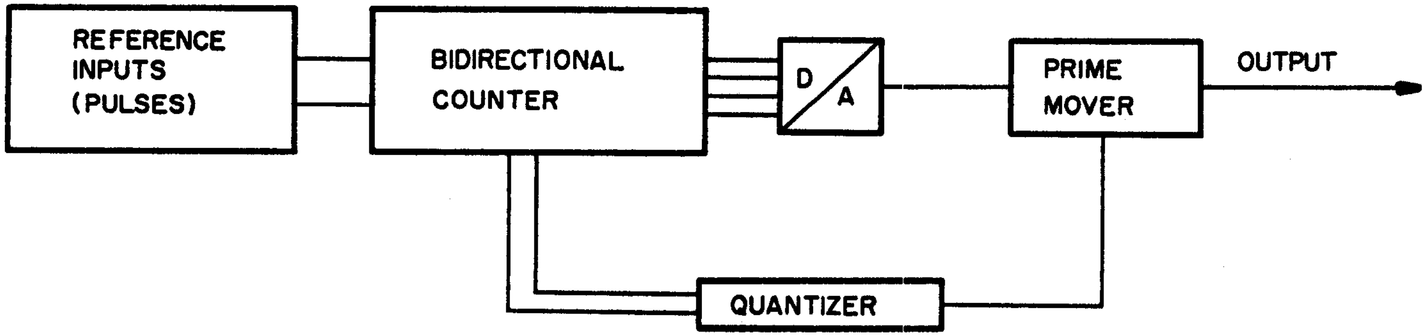


FIGURE 1 TYPICAL INCREMENTAL DIGITAL CONTROL LOOP

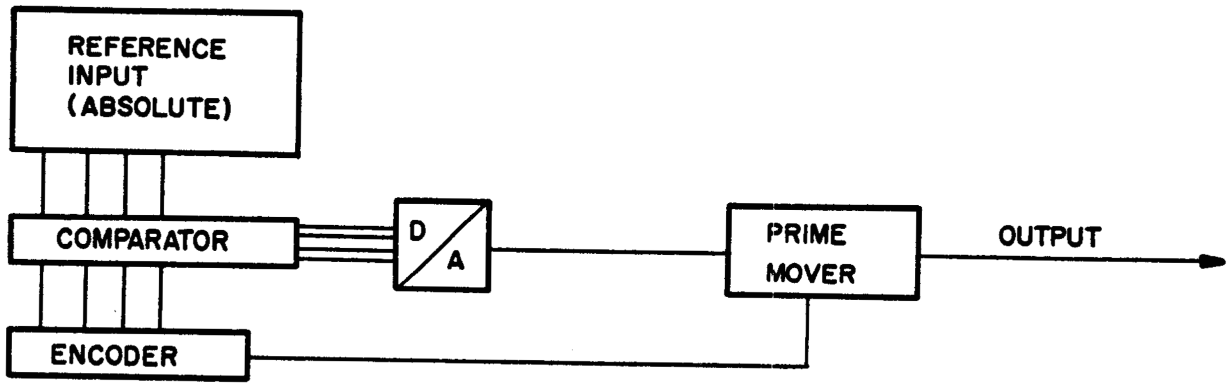


FIGURE 2 TYPICAL ABSOLUTE DIGITAL CONTROL SYSTEM

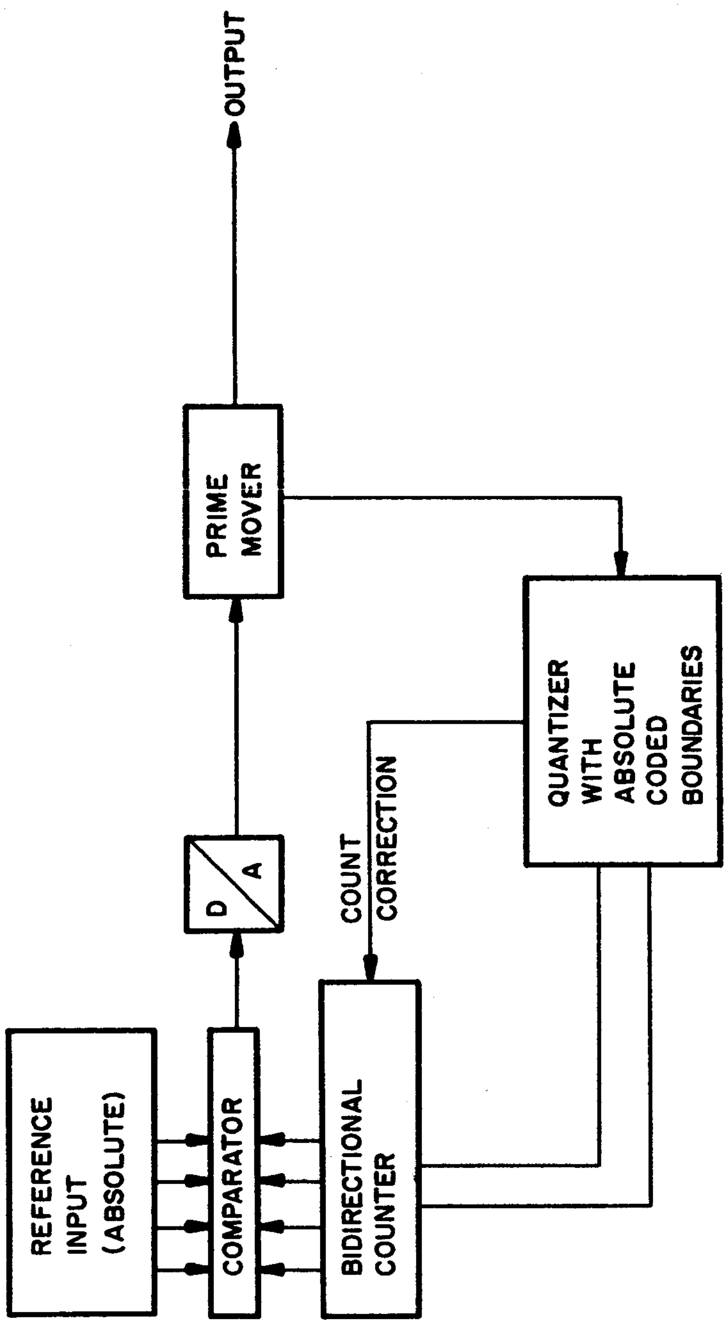


FIGURE 3 SPECIAL INCREMENTAL /ABSOLUTE DIGITAL CONTROL LOOP

CHAPTER II
SYSTEM DESCRIPTION

Pulse Rate Measurement Technique

Two basic methods may be employed to determine the rate of a train of pulses. The first method involves the measurement of the time interval between each pulse. The result would represent the instantaneous value; however, the value of this interval is inversely proportional to the rate. Consequently, a non-linear device or some type of complex gating or conversion scheme would be necessary to provide a value directly proportional to the rate.

The second general method requires the sampling of the number of pulses occurring during constant time intervals. The result in this case represents the average rate during the sampling period. In a conventional digital system using this method, the sampling times are independent and consecutive, and a new value of the average pulse rate appears only at the conclusion of each sampling period. Unfortunately, since a new value of rate appears only once for each sampling interval, any appreciable change in the pulse rate will not be detected until the end of the sample.

The proposed method of measuring the rate of a pulse train will provide a continuous indication of the pulse rate averaged over a selected time interval.[2*] This device utilizes a shift register containing n_{sh} bits and a bidirectional counter (BDC) as shown in Figure 4. The rate is determined as described below.

The contents of the shift register will be shifted at a constant frequency, f_{sh}; therefore, a delay, τ, will exist between the information input and the output of the register where

$$\tau = \frac{n_{sh}}{f_{sh}} \tag{1}$$

The input pulse train, in which the minimum time between pulses is f_i^{-1}, enters the input of the shift register. If $f_{sh} > f_i$, the number of pulses in the shift register, P_{sh}, will simply be the number of pulses which entered the system during the past τ seconds. The average input pulse rate is, therefore,

$$R_{av} = \frac{N_{sh}}{\tau} \tag{2}$$

Since each input pulse enters the shift register immediately (disregarding synchronization delays) and as the contents are shifted at a constant frequency, the average pulse rate

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*The superscript numerals refer to the Bibliography.

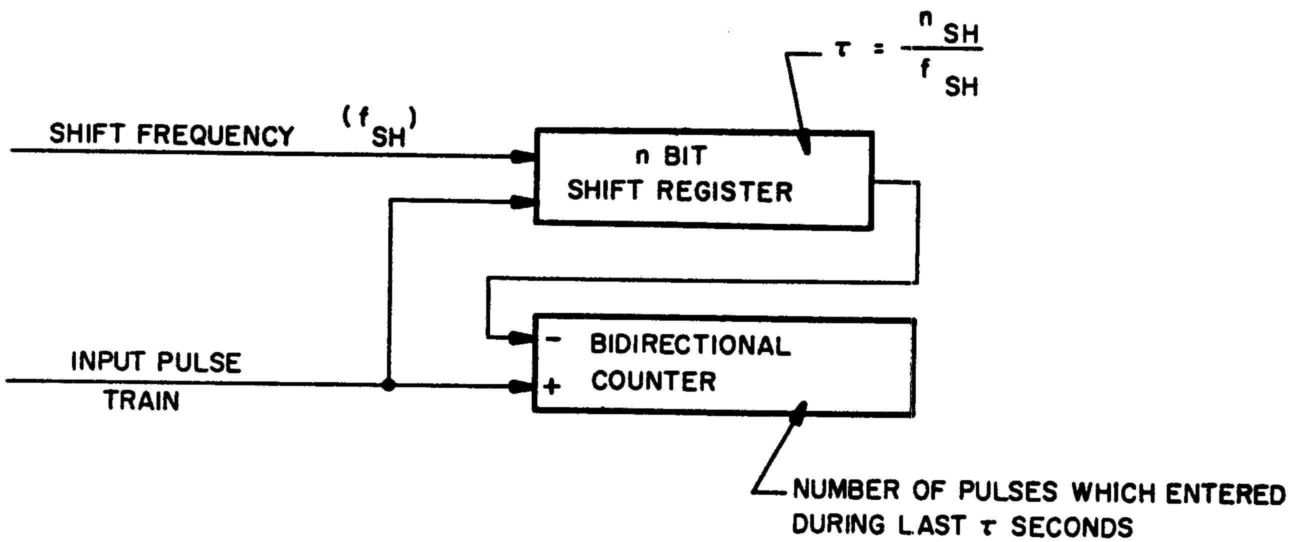


FIGURE 4 PROPOSED PULSE RATE MEASUREMENT TECHNIQUE

will be continuously monitored and any change in the input pulse rate will have an immediate effect on the value of R_{av}.

The bidirectional counter is employed to indicate the number of pulses which are in the shift register. In order to accomplish this task, the input pulses which enter the shift register are also used to increase the value of the BDC, and the pulses which exit from the shift register are used to decrease the value of the BDC. The value of the counter, at time T is then R_T, where

$$R_T = \sum_{t=0}^{T} P_t - \sum_{t=0}^{T-\tau} P_t = \sum_{t=T-\tau}^{T} P_t = N_{sh} \qquad (3)$$

Thus from equations (2) and (3),

$$R_T = (R_{av}) \ (\tau) \qquad (4)$$

and the value of the counter will be directly proportional to the average input pulse rate.

Note that in the measurement of the output pulse rate of a quantizer, if both the distance between the quanta boundaries and τ approach zero, in the limit this would reduce to the first derivative which is an exact representation of the rate.

To employ this technique in an actual system, it is necessary to consider the effects of the parameters τ, n_{sh}, and f_{sh}. If τ is smaller than the minimum time between the input pulses, the indicated value of the rate will never be greater than

1 and it will normally be 0; therefore, it is necessary for τ to be long enough to allow variations in the pulse rate to be detected. On the other hand, if τ is too long, the indicated value of the rate and the average value may be entirely different. An example of this case is when the input pulse rate varies sinusoidally and τ is equal to the time of the period--in such a case, the indicated rate would be a constant and the information would be meaningless.

It is thus desirable to select a value of τ first. This should be chosen to allow a balance between the upper and lower boundary effect. If it is not possible to stay within a reasonable limit, it will be necessary to modify the significance of the input pulse rate, if possible. For example, in a digital positioning servosystem, it is necessary to increase the number of quanta zones in the unit measuring the output variable. Once τ has been determined, a value of n_{sh} may be selected which will accommodate the maximum number of pulses which might occur during the delay period. A value of f_{sh} is then selected which will satisfy Equation (1).

System Application

In a servosystem which is used to control the position of the output, a feedback loop is used to indicate the difference between the reference and output positions; moreover, it is quite often necessary to provide a second feedback loop. This extra loop is necessary when either the gain is too large or the time lag of the system is so long that the output will be unstable or will oscillate considerably before reaching the desired position. In the second feedback loop, a signal is generated which is proportional to the output rate, and this signal is used to oppose the motion of the output. As a result, the rapid velocities of the output will be reduced and this in turn will reduce the overshoot. Thus, the effective time constants are reduced and the damping coefficient is increased.

In most digital positioning systems of this type, analog techniques have formerly been used to generate rate feedback; however, the pulse rate measurement technique will be used instead, and its effectiveness will be determined by analyzing the effect on the damping coefficient.

Experimental System Description

A general block diagram of the system is shown in Figure 5.

The quantizer is used to measure the output, $\theta_o(t)$, of the controlled variable. Each time the output rotates a specified angular distance or quantum, a pulse will be emitted on either a "forward" or a "backward" line indicating that a quanta boundary has been traversed and the corresponding direction of rotation.

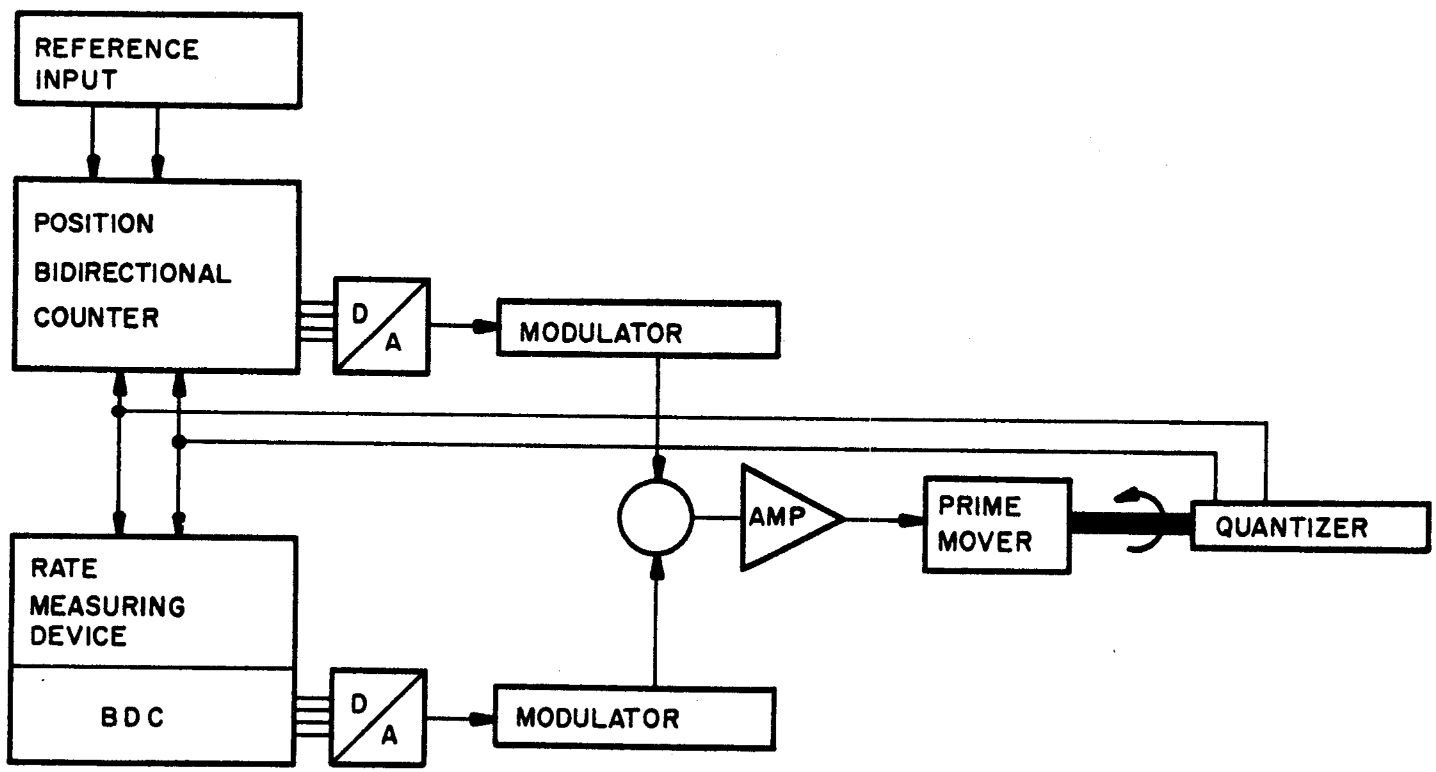


FIGURE 5 GENERAL SYSTEM BLOCK DIAGRAM

The "position" bidirectional counter will indicate the sign and magnitude of the difference between the desired (reference) change and the number of quanta boundaries the output has actually crossed.

The "rate" bidirectional counter utilizes the rate measurement technique described above and indicates the average rate of the pulses emitted by the quantizer. The sign is used to indicate the average direction of rotation.

The signals from the two bidirectional counters are each converted to a D.C. voltage which is then modulated. The A. C. voltages are summed and amplified in order to drive an A. C. servomotor which serves as the prime mover.

CHAPTER III
MATHEMATICAL MODEL

Introduction

In order to evaluate the performance of the experimental system, a mathematical model is developed. This can be used to analyze the theoretical performance and compare it with the actual system response.

Although the servo amplifier, motor, and gear train operate in a continuous mode, a discontinuity in the control is introduced by the quantizer, bidirectional counter, and pulse synchronization unit. As a result, it is impossible to utilize a simple mathematical relationship to describe the output response.

The model developed for the system is shown in Figure 6 and adapted from a similar analysis by C. K. Taft.[5] This type of model is presented since it provides an exact description of the signal flow in both the continuous and discontinuous portions of the system.

Reference Input

The reference input will consist of a train of pulses indicating a change of onequantum or itwill be a larger step input applied directly to the position counter; thus, the input pulses,

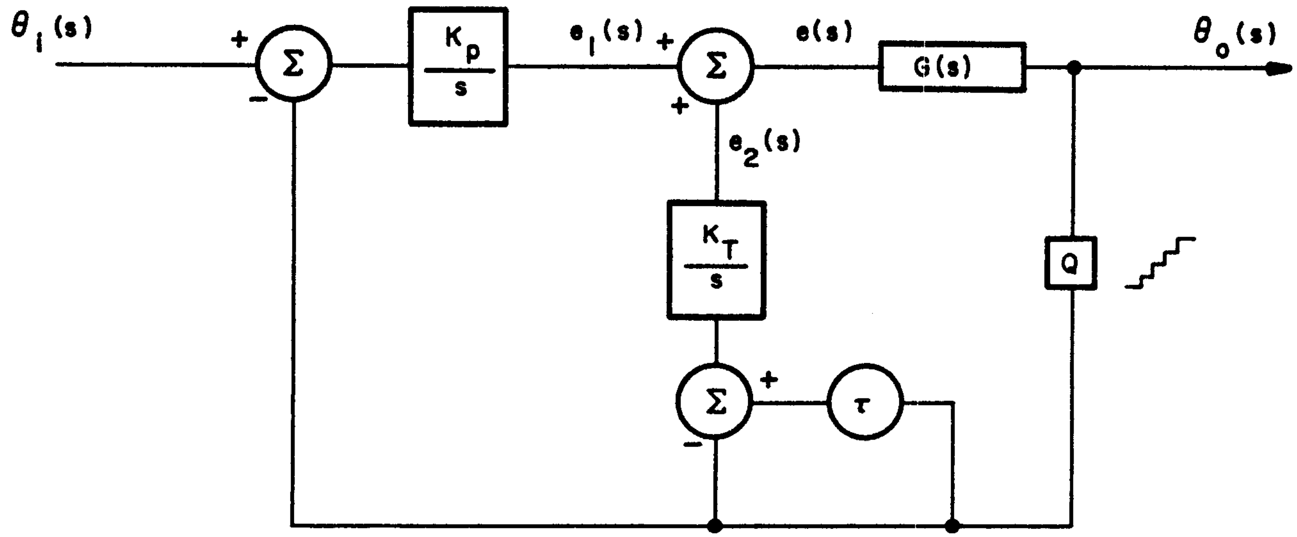


FIGURE 6 MATHEMATICAL MODEL

$\theta_i(t)$, can be represented by

$$\theta_i(t) = \sum_{n=0}^{\infty} A_n \, \delta\,(t - T_{rn}) \tag{5}$$

where $\delta\,(t - T_{rn})$ is a unit impulse representing the n^{th} pulse which occurs at time $t = T_{rn}$. If the input is a train of pulses, $A_n = \pm 1$ dependent upon which input line the pulse train enters. In the case of the direct input, $T_{rn} = 0$, $A_n = 0$ for all $n > 0$, and A_o is an integer representing a step input of A_o quanta.

Feedback Pulses

The feedback pulses are produced by the quantizer unit. Whenever the output pulse passes a "quanta" boundary, a pulse is emitted on one of two lines indicating both the time and direction of crossing. The quantizer output pulses will be represented by $f(t)$, with

$$f(t) = \sum_{m=0}^{\infty} B_m \, \delta\,(t - T_{fm}) \tag{6}$$

where $\delta\,(t - T_{fm})$ is a unit impulse due to the m^{th} quanta boundary traversed at time $t = T_{fm}$ and $B_m = \pm 1$ depending on the direction of rotation of the output.

Position Feedback

The value of the position counter will be

$$e_1(t) = K_p \int_o^t [\theta_i(t) - f(t)] \; dt \tag{7}$$

thus from equations (5), (6), and (7),

$$e_1(t) = K_p \int_o^t \left[\sum_{n=0}^{\infty} A_n \, \delta(t - T_{rn}) - \sum_{m=0}^{\infty} B_m \, \delta(t - T_{fm}) \right] dt \tag{8}$$

The bidirectional counter, D/A converter, and phase controlled modulator are, therefore, represented by a summing junction, integrator, and gain.

Rate Feedback

The value of the rate counter will be

$$e_2(t) = K_T \int_o^t [- f(t) + f(t - \tau)] \, dt \tag{9}$$

where

τ = delay time for all t > shift register delay time

τ = 0 for all t < shift register delay time

From equations (6) and (9),

$$e_2(t) = K_T \int_o^t \left[\sum_{m=0}^{\infty} B_m \, \delta(t - \tau - T_{fm}) - B_m \, \delta(t - T_{fm}) \right] dt \tag{10}$$

The bidirectional counter, D/A converter, and phase controlled modulator are represented by a summing junction, integrator, and gain, and the shift register is represented by a time delay.

Total Feedback

The signals from the rate and position loops are then summed, resulting in a signal $e(t)$, where

$$e(t) = e_1(t) + e_2(t)$$

$$e(t) = \int_o^t \left[\sum_{m=0}^{\infty} B_m \left[- (K_p + K_T) \delta(t - T_{fm}) + K_T \delta(t - \tau - T_{fm}) \right] + \sum_{n=0}^{\infty} A_n K_p \delta(t - T_{rn}) \right] dt \qquad (11)$$

Since the integral of a unit impulse is simply a step input,

$$\int_o^t \delta(t - T)\, dt = u(t - T) \qquad (12)$$

Thus,

$$e(t) = \sum_{m=0}^{\infty} B_m \left[- (K_p + K_T) u(t - T_{fm}) + K_T u(t - \tau - T_{fm}) \right] + \sum_{n=0}^{\infty} K_p A_n u(t - T_{fm}) \qquad (13)$$

The input into the continuous portion of the system is then $e(t)$, which is simply a series of unit steps.

Response of Continuous Control Section

It is now possible to determine the output, $\theta_o(t)$, using a continuous system approach by considering the input to $G(s)$

as a series of step inputs. Using a time domain approach,

$$\theta_o(s) = G(s) \;\; e(s) \tag{14}$$

From equation (13), the Laplace transform of e(t) is

$$e(s) = \sum_{m=0}^{\infty} B_m \left[\frac{-(K_p + K_T)\, e^{-T_{fm}}}{s} + \frac{K_T\, e^{-\tau - T_{fm}}}{s} \right]$$

$$+ \sum_{n=0}^{\infty} A_n K_p \frac{e^{-T_{rn}}}{s} \tag{15}$$

Therefore, from equations (14) and (15),

$$\theta_o(s) = \frac{G(s)}{s} \left[\sum_{m=0}^{\infty} B_m \left[-(K_p + K_T)\, e^{-T_{fm}} + K_T\, e^{-\tau - T_{fm}} \right] \right.$$

$$\left. + \sum_{n=0}^{\infty} A_n K_p \, e^{-T_{rn}} \right] \tag{16}$$

A quantity, q(t), is defined where

$$q(t) = \mathcal{L}^{-1} \frac{G(s)}{s} \tag{17}$$

which is the response of the continuous portion of the system to a step input with zero initial conditions. Then, from equations (16) and (17), the output is

$$\theta_o(t) = \mathcal{L}^{-1} \left(\theta_o(s)\right)$$

$$\theta_o(t) = \sum_{m=0}^{\infty} B_m \left[-(K_p + K_T)\, q(t - T_{fm})\, u(t - T_{fm}) + K_T\, q(t - \tau - T_{fm})\, u(t - \tau - T_{fm}) \right] + \sum_{n=0}^{\infty} K_p A_n\, q(t - T_{rn})\, u(t - T_{rn}) \tag{18}$$

In this case, $q(t - T)\, u(t - T)$ is the unit step response delayed in time by $t = T$.

The value of $\theta_o(t)$ determined from equation (18) represents the combination of effects of both the continuous and discontinuous portions of the system. The crossing of a quantum point then occurs when the value of $\theta_o(t)$ increases or decreases by one quantum from the previous crossing or from the initial starting point.

The continuous portion of the system, $G(s)$, represents the effects of the servo amplifier, motor, and gear train. The transfer function will be

$$G(s) = \frac{K}{s(1 + T_L s)\,(1 + T_M s)} \tag{19}$$

where

K = amplifier gear train gain

T_L = time constant due to motor and gear train inertia

T_M = time constant due to electrical characteristics of the motor.

The response of this will then be

$$q(t) = \sum_{\substack{\text{poles} \\ \text{of } G(s)}} \text{residues} \left[\frac{K e^{st}}{T_M T_L s^2 (s + \frac{1}{T_M})(s + \frac{1}{T_L})} \right] \tag{20}$$

$$q(t) = \left[\frac{K e^{st}}{T_M T_L s^2 (s + \frac{1}{T_L})} \right]_{s = -\frac{1}{T_M}} + \left[\frac{K e^{st}}{T_M T_L s^2 (s + \frac{1}{T_M})} \right]_{s = -\frac{1}{T_L}}$$

$$+ \frac{d}{ds} \left[\frac{K e^{st}}{T_M T_L (s + \frac{1}{T_L})(s + \frac{1}{T_M})} \right]_{s = 0} \tag{21}$$

$$q(t) = K \left[\frac{T_M^2 e^{-\frac{t}{T_M}}}{T_M - T_L} + \frac{T_L^2 e^{-\frac{t}{T_L}}}{T_L - T_M} + t - (T_M + T_L) \right] \tag{22}$$

If $T_L >> T_M$, this reduces to

$$q(t) \simeq K \left[T_L e^{-\frac{t}{T_L}} + t - T_L \right] \tag{23}$$

Discussion

Using equations (18) and (23), therefore, it is possible to determine the output response to either a large step change or a series of unit step changes. In the analysis of the proposed system, the value of T_L will be dependent upon the **servomotor** and load and a value of K_p will be selected which will allow an investigation of the effect of the tachometer feedback on the damping. A step input will be applied to the system and the response will be determined for various values of K_T and τ.

Note that equation (18) consists of a series of step responses starting at times T_{rn}, T_{fm}, and $T_{fm} + \tau$. The times T_{rn} and the delay τ will be given; however, it is impossible to predict the times at which the feedback pulses occur. As a result, the solution becomes rather complex and it is necessary to compute both the output response and the times of T_{fm} simultaneously. A graphical technique for accomplishing this is suggestedby C. K. Taft;[5] however, because of the time and inaccuracies involved in this method, another method was adopted. A computer program was written in Algol for use on the Burroughs 220 computer and was later modified for use on the Univac 1107 because of the time required for each set of computations. The program and flow diagram are described in Appendix II and the results are given in the next section.

CHAPTER IV
SYSTEM ANALYSIS

Selection of Parameters K_p and T_L

The desired effect of the addition of rate feedback in a positioning servosystem is to reduce oscillations by increasing the effective time constant of the system which in turn increases both the damping coefficient and natural frequency. In the experimental system, therefore, it is necessary to select a set of parameters which will cause the system to oscillate when no rate feedback is present. This will then allow the effect of the rate feedback to be observed. In order to investigate the influence of K_p and T_L on the output, the closed loop system is approximated by a continuous system.

Referring to equation (21), the transfer function of the system with both rate and position feedback, neglecting T_m, will be approximately

$$G(s) = \frac{1}{s(1 + T_L s)} (K_p + K_T s) \tag{24}$$

The input-output relationship is

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{G(s)}{1 + G(s)} \tag{25}$$

Therefore,

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{K_p + K_T s}{s(1 + T_L s) + K_p + K_T s} \tag{26}$$

If no rate feedback is used, $K_T = 0$, and rearranging time,

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{\left(\frac{K_p}{T_L}\right)}{s^2 + \left(\frac{1}{T_L}\right) s + \left(\frac{K_p}{T_L}\right)} \tag{27}$$

The general equation of a second order system is usually presented as

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{\omega_n^2}{s^2 + (2\zeta\omega_n)s + \omega_n^2} \tag{28}$$

where ω_n = natural frequency

ζ = damping coefficient.

If equations (27) and (28) are compared, it is seen that

$$\omega_n = \sqrt{\frac{K_p}{T_L}} \tag{29}$$

$$\zeta = \frac{1}{2 T_L \omega_n} = \frac{1}{2\sqrt{T_L K_p}} \tag{30}$$

Solving equation (28), the response to a unit step input with $-1 < \zeta < +1$ will be,

$$\theta_o(t) = \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin \left[(\omega_n \sqrt{1 - \zeta^2}) t + \alpha \right] - 1 \tag{31}$$

where α is determined by the initial conditions.

The response is thus a series of oscillations which decay exponentially. The major effect of changing the damping coefficient is thus to increase the decay time. In terms of the experimental system parameters, equation (31) may be rewritten as

$$\theta_o(t) = \frac{1}{\sqrt{1 - \frac{1}{4T_L K_p}}} e^{-\frac{t}{2T_L}} \sin\left[\frac{1}{\sqrt{K_p}} \sqrt{\left(T_L - \frac{1}{4K_p}\right)} t + \alpha\right] - 1 \tag{32}$$

It may be seen from equation (32) that the rate of decay is controlled primarily by T_L and that the frequency of oscillation is controlled by both K_p and T_L.

In the experimental system, the value of T_L is determined from the motor speed-torque characteristics and the inertia of the motor and load.

An approximate value of T_L was calculated to be $T_L = 0.125$ sec. Using equation (33), the relation between K_p and ζ is then as follows:

$\zeta = 0.1$ $\quad K_p = 200$

$\zeta = 0.2$ $\quad K_p = 50$

$\zeta = 0.3$ $\quad K_p = 22$

$\zeta = 0.4$ $\quad K_p = 12.5$

A value of $K_p = 25$ was determined to be an adequate value as it will allow a little oscillation if $K_T = 0$ and is also compatible with the experimental model. The theoretical response of this system to a step input of 32 quanta is shown in Figure 7.

With $K_p \cong 25 \text{ sec}^{-1}$ and $T_L \cong 0.125$ sec, the effects of the rate measurement were investigated by first studying the effect of both K_T and τ predicted by the mathematical model and then comparing these results with the actual system performance. As a further test, the response of the system using the digital tachometer was compared with the response using an analog tachometer.

Computer Analysis of Effects of Parameters K_T and τ

The computer program which is described in Appendix II was used to analyze the effects of K_T and τ on the system response to a step input of 32 quanta and using the parameters for K_p and T_L selected in the preceeding section.

The results of the study are shown in Figures 8 and 9. Several conclusions may be drawn from these results. First, they indicate that, theoretically, the proposed rate technique

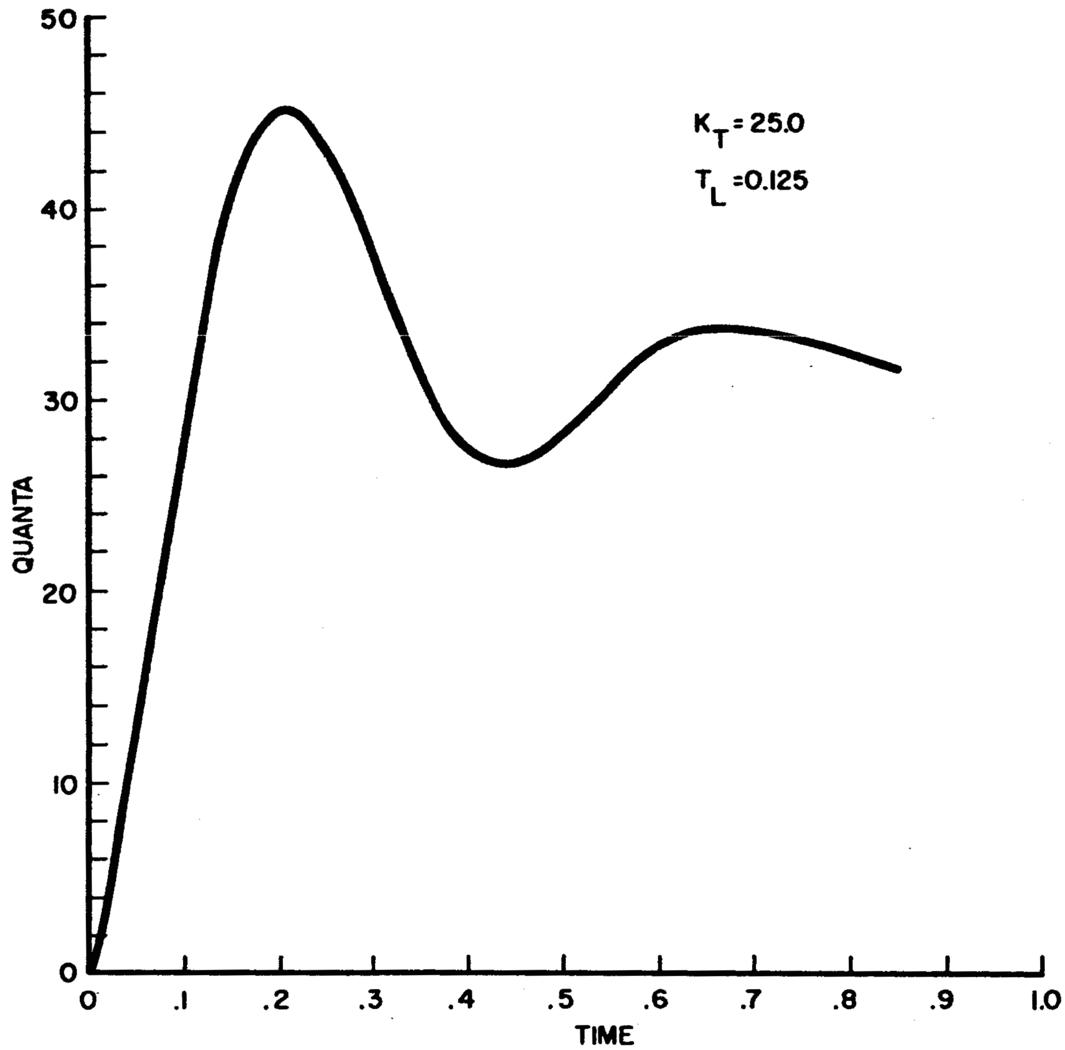


FIGURE 7 PREDICTED RESPONSE USING CONTINUOUS SYSTEM APPROXIMATION

will be useful as a digital compensation device. Second, both K_p and τ have a similar effect on the effective damping coefficient and as a result, the damping can be increased by increasing either K_p or τ or both. Third, these results may be used to select the parameters to be used in the experimental system. For example, the value of τ can be used with the results to determine the maximum number of quanta boundaries crossed during the delay time which will indicate the number of bits to be used in the shift registers and which in turn may be used to determine the shift frequency.

Experimental Results

A step input of 32 quanta was applied to the experimental system. In the model, $T_L = 0.125$ sec, $K_p = 25\ sec^{-1}$, $n_{sh} = 14$ bits, and K_T and f_{sh} were adjusted to determine the response as a function of the tachometer gain and delay time. These results appear in Figures 10-12. It is then possible to compare the results of the case in which $K_T = 0$ with the results predicted both by the mathematical model study and by the continuous system approximation; furthermore, the experimental system response for several different values of K_T and τ, Figures 11 and 12, may also be compared with the results predicted in the computer study, Figures 8 and 9. The correlation of these results appears to be well within the bounds of experimental error.

As a further investigation of the performance of the system, the tachometer winding on the servomotor was connected in the feedback loop and the digital tachometer was disconnected. The output response to a 32 quanta step input was measured using several values of tachometer gain, and the results are shown in Figure 13. These results are nearly identical to the response using the digital tachometer (Figures 11 and 12).

Conclusions

A technique which allows the "continuous" measurement of the average pulse rate was used to provide rate feedback in an incremental digital servosystem. The response of the experimental system was compared with the predicted response from the mathematical model of the system. The experimental system response was also compared with that of the same system with the exception that an analog tachometer was used in place of the digital tachometer.

The high degree of correlation between the predicted and experimental responses indicate that the rate measurement technique is a very effective tool as a digital compensation device. Although this was investigated only in a feedback loop, this indicates that it may be possible to use it in a feedforward loop and as a part of a self-adaptive control system.

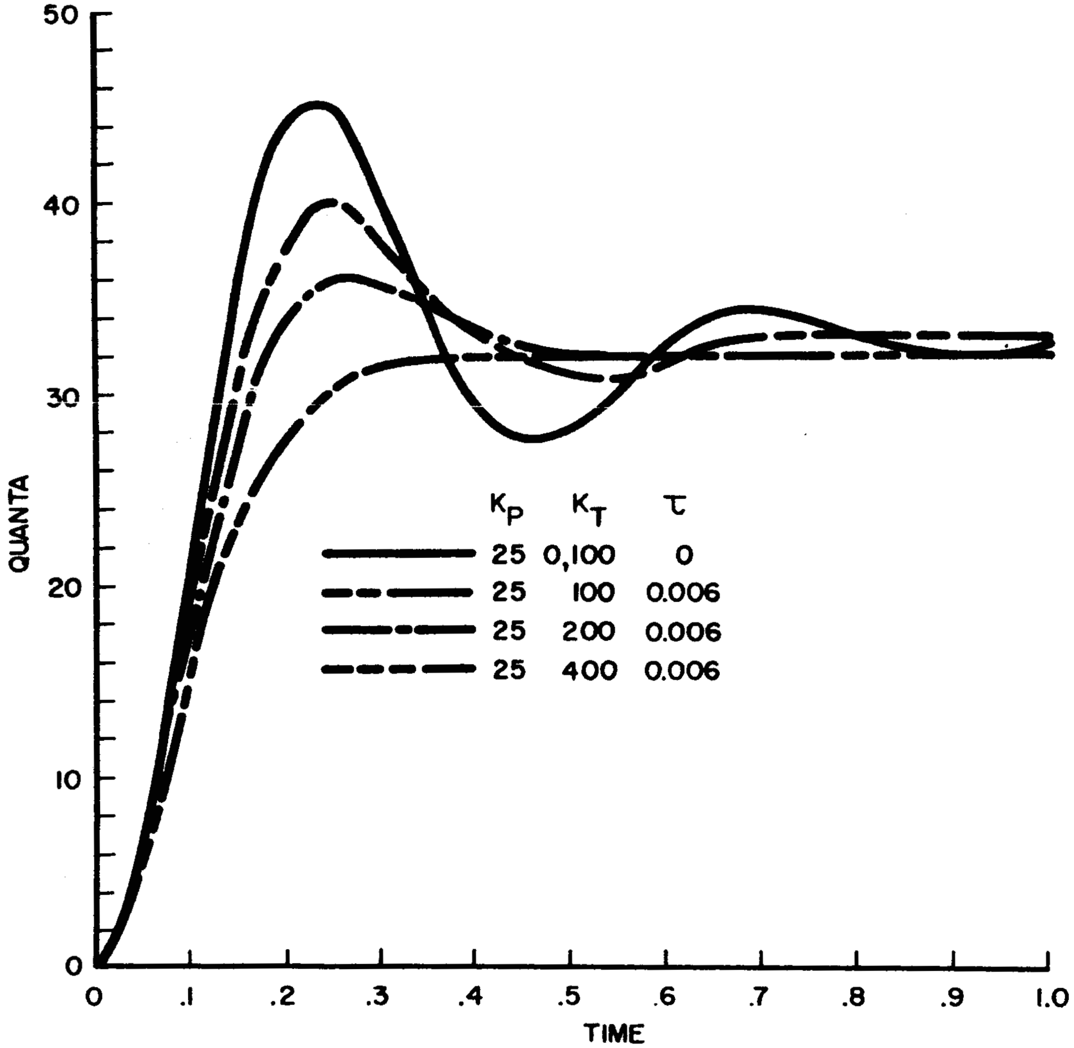


FIGURE 8 THEORETICAL RESPONSE AS A FUNCTION OF K_T WITH τ CONSTANT

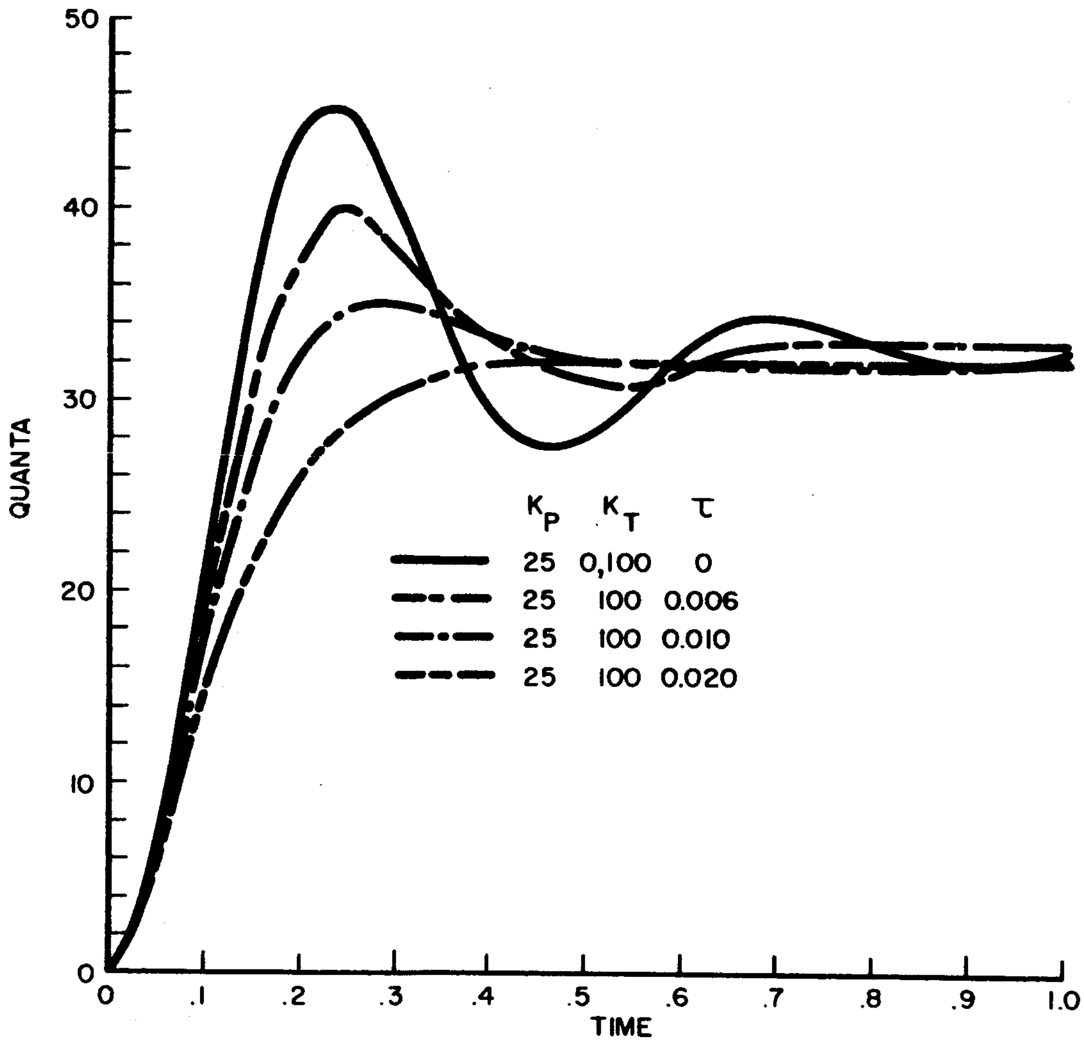
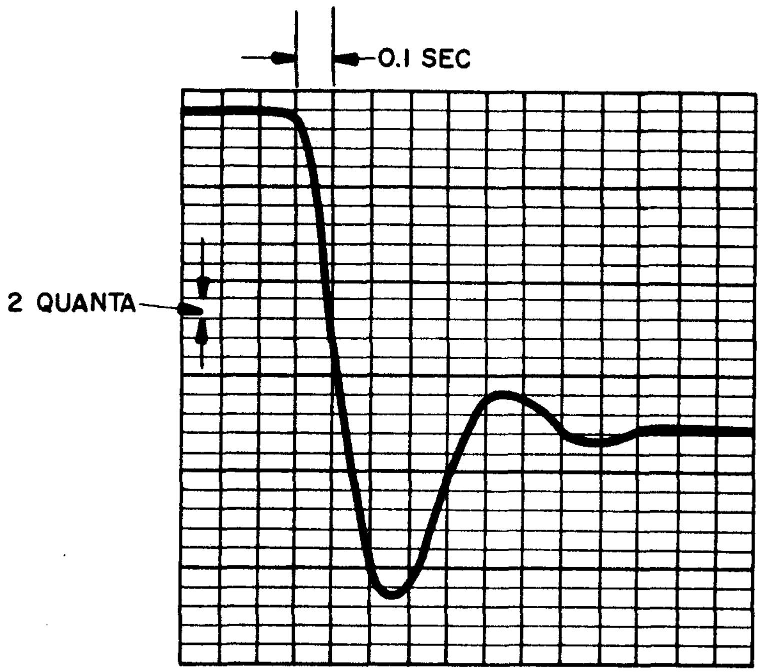


FIGURE 9 THEORETICAL RESPONSE AS A FUNCTION OF τ WITH K_T CONSTANT

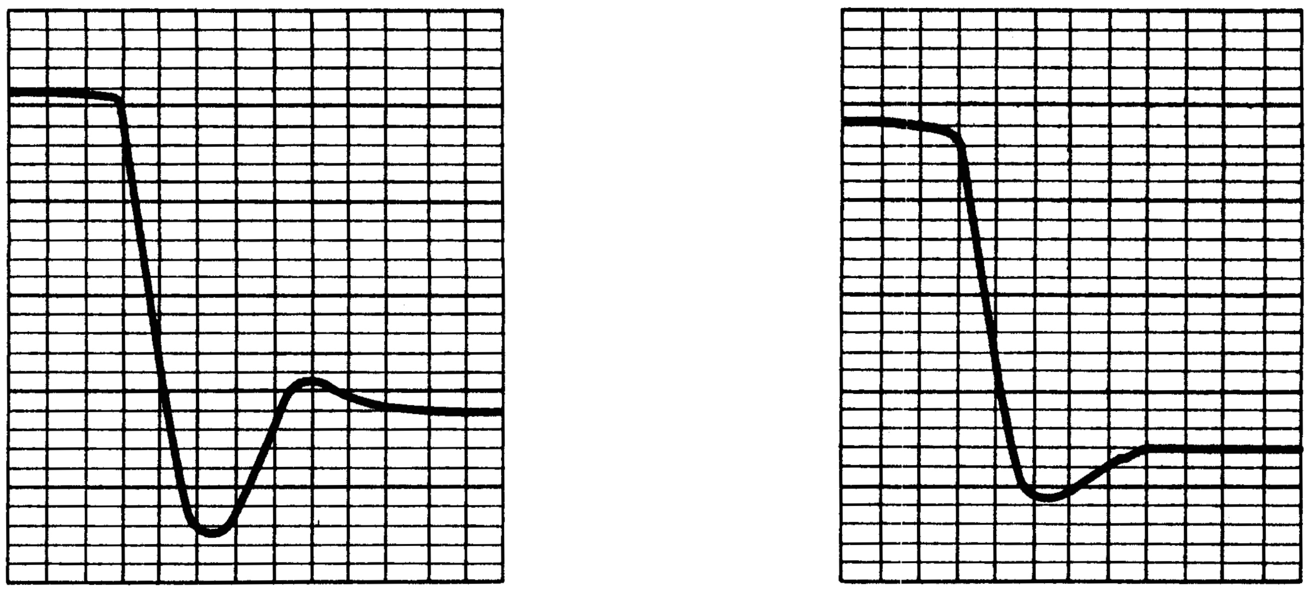


$K_p = 25$

$K_T = 0,100$

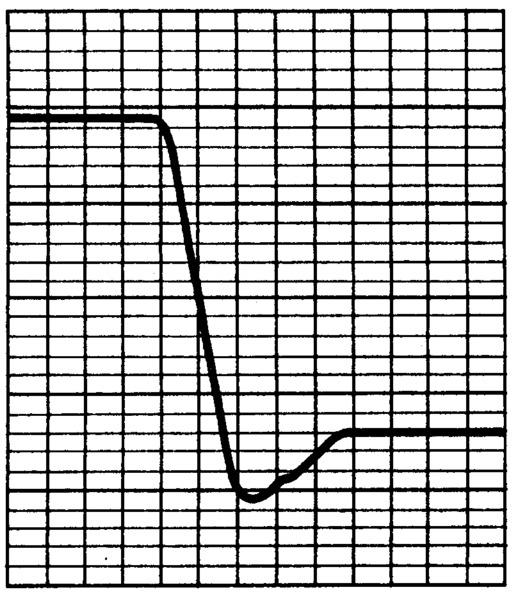
$\tau = 0$

FIGURE 10 RESPONSE WITH NO RATE FEEDBACK

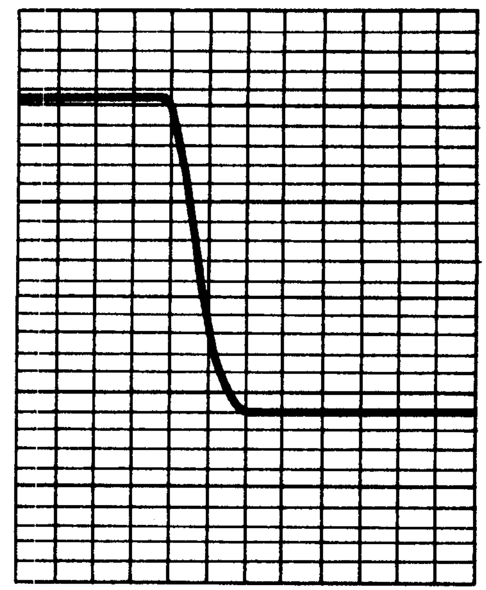


$K_T = 100$ $\tau = 0.006$ $K_T = 200$ $\tau = 0.006$

FIGURE 11 RESPONSE USING DIGITAL TACHOMETER



(a)
$K_T=100$ $\tau=0.014$ SEC



(b)
$K_T=100$ $\tau=0.028$ SEC

FIGURE 12 RESPONSE USING DIGITAL TACHOMETER

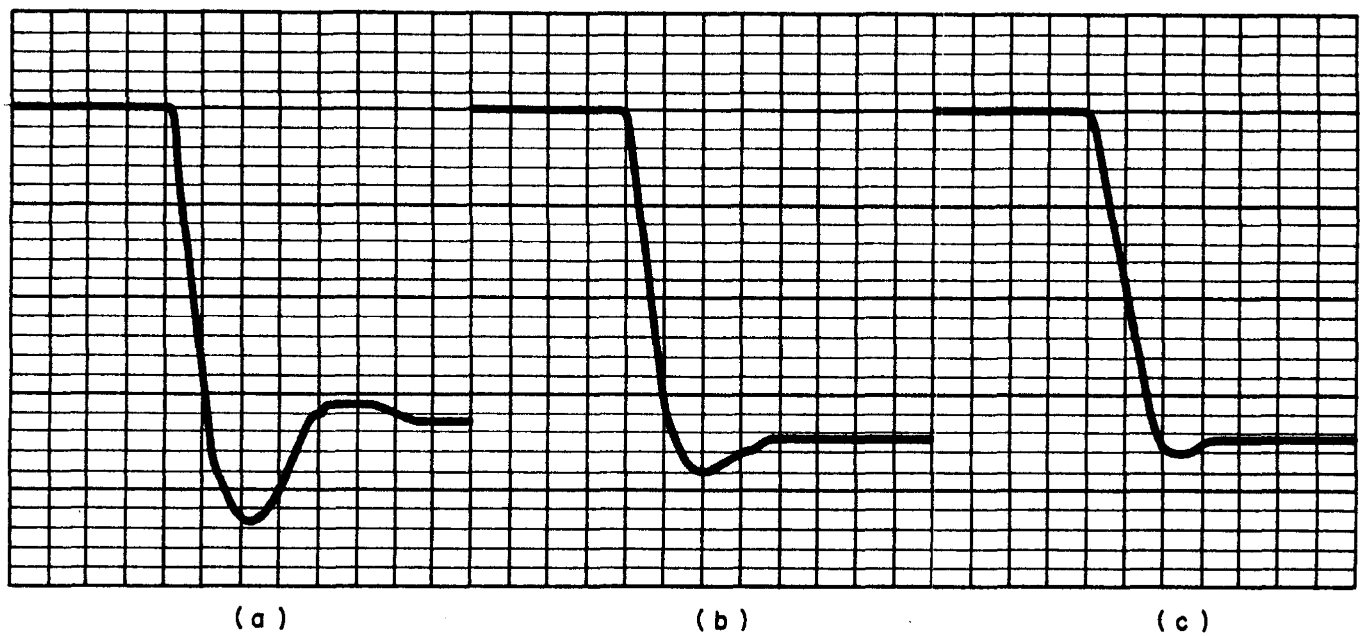


FIGURE 13 RESPONSE USING ANALOG TACHOMETER

APPENDIX I
DETAILED SYSTEM DESIGN

Introduction

The incremental digital system which is analyzed in this report is described in detail in this section. The system is comprised of the following basic units:

1. Quantizer, wave shaping, and direction sensing logic.
2. Synchronization and timing logic.
3. Rate and position bidirectional counters.
4. Shift registers.
5. Digital to analog converter.
6. Phase controlled modulator.
7. Servo amplifier, motor and gear train.

The interconnection of the above units is shown in Figure 14, and a photograph of the system is shown in Figure 15.

The logical designs of units 2, 3, and 4, which perform the major digital operations were implemented using the "Digital Synthesizer". This unit, shown in Figure 15, has been recently constructed in the Engineering Design Center of Case Institute of Technology and consists of a variety of Wang Laboratories' Series 200 Logibloc

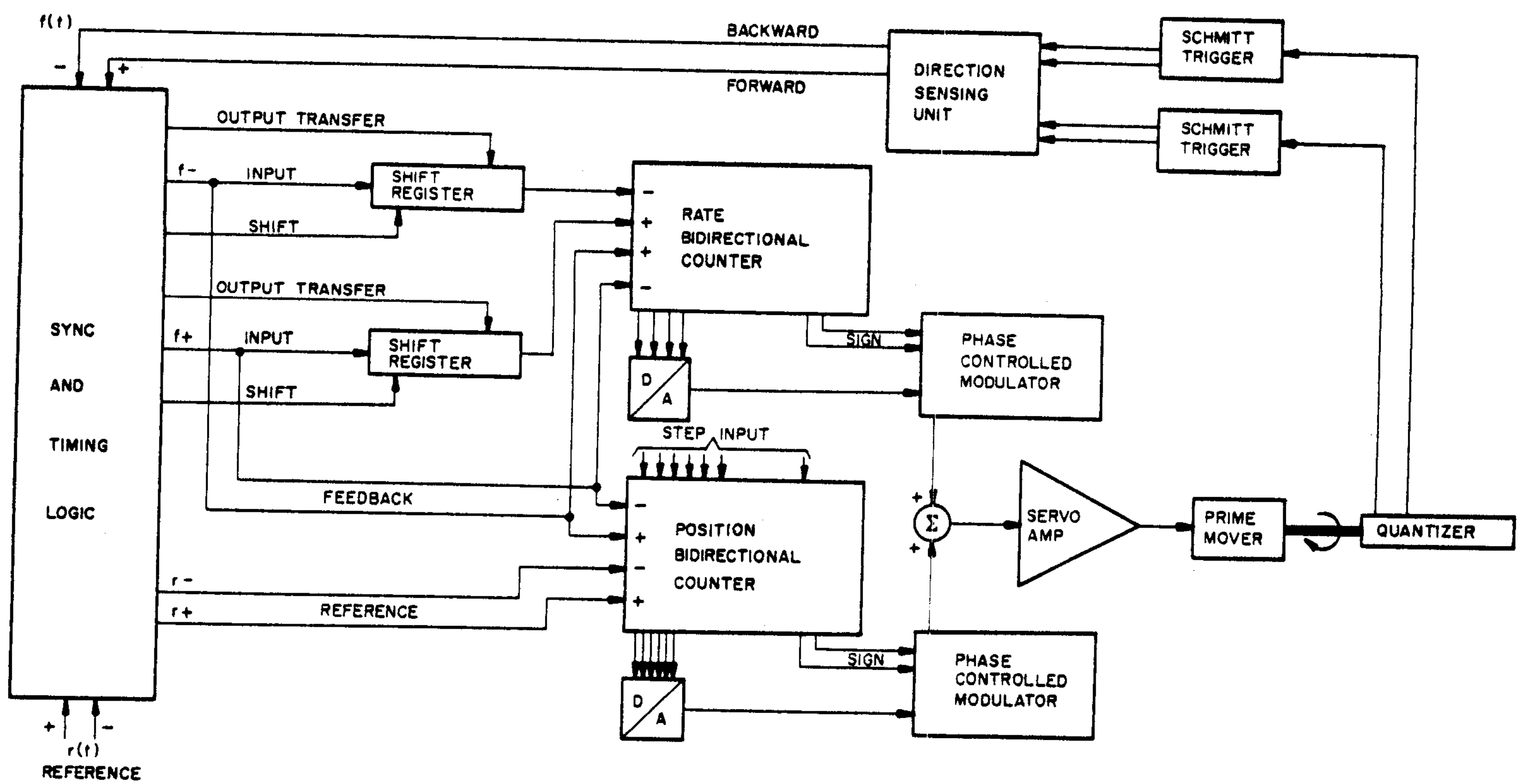


FIGURE 14 DETAILED BLOCK DIAGRAM OF EXPERIMENTAL SYSTEM

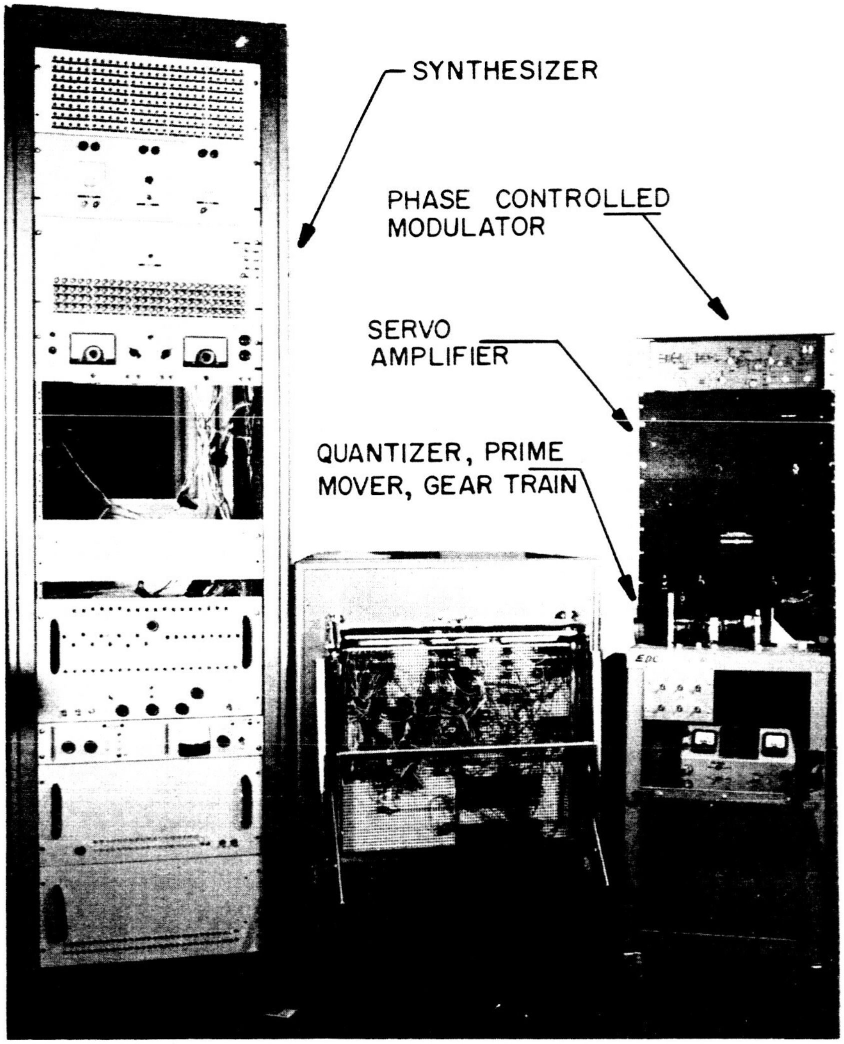


FIGURE 15 EXPERIMENTAL SYSTEM

Transistorized Digital Modules which are shown in Figures 16 and 17. To allow this unit to be used for a wide range of applications, 100 flip-flops, 80 - 1 input NOR gates, 80 - 2 input NOR gates, 30 - 3 input NOR gates, 20 - 5 input NOR gates, 100 gated pulse generators, 200 steering gates, and 20 one shot multivibrators are all wired to a central patchboard panel. The user may then connect them in any desired manner to implement any logical operation within the prescribed limits. Twenty external card sockets are also provided to enable the user to use special logic cards. The use of this unit further allows many of the logical designs to be minimized and tested for reliability before using them as an integral part of the experimental system.

<u>Quantizer</u>

The quantizer selected for use in the system is a Wang Laboratories, Model 43-1000 incremental encoder which will produce 1000 equally spaced impulses for each revolution of the encoder, or one pulse for each angular displacement of 0.36°. This device consists of a photographic disc with 250 pairs of clear and opaque lines spaced equally near the edge. Two pairs of lamps and light sensitive diodes are spaced a distance of $360\left(\frac{n}{250} + \frac{1}{1000}\right)$ degrees apart (n is a positive integer) and they are used to increase

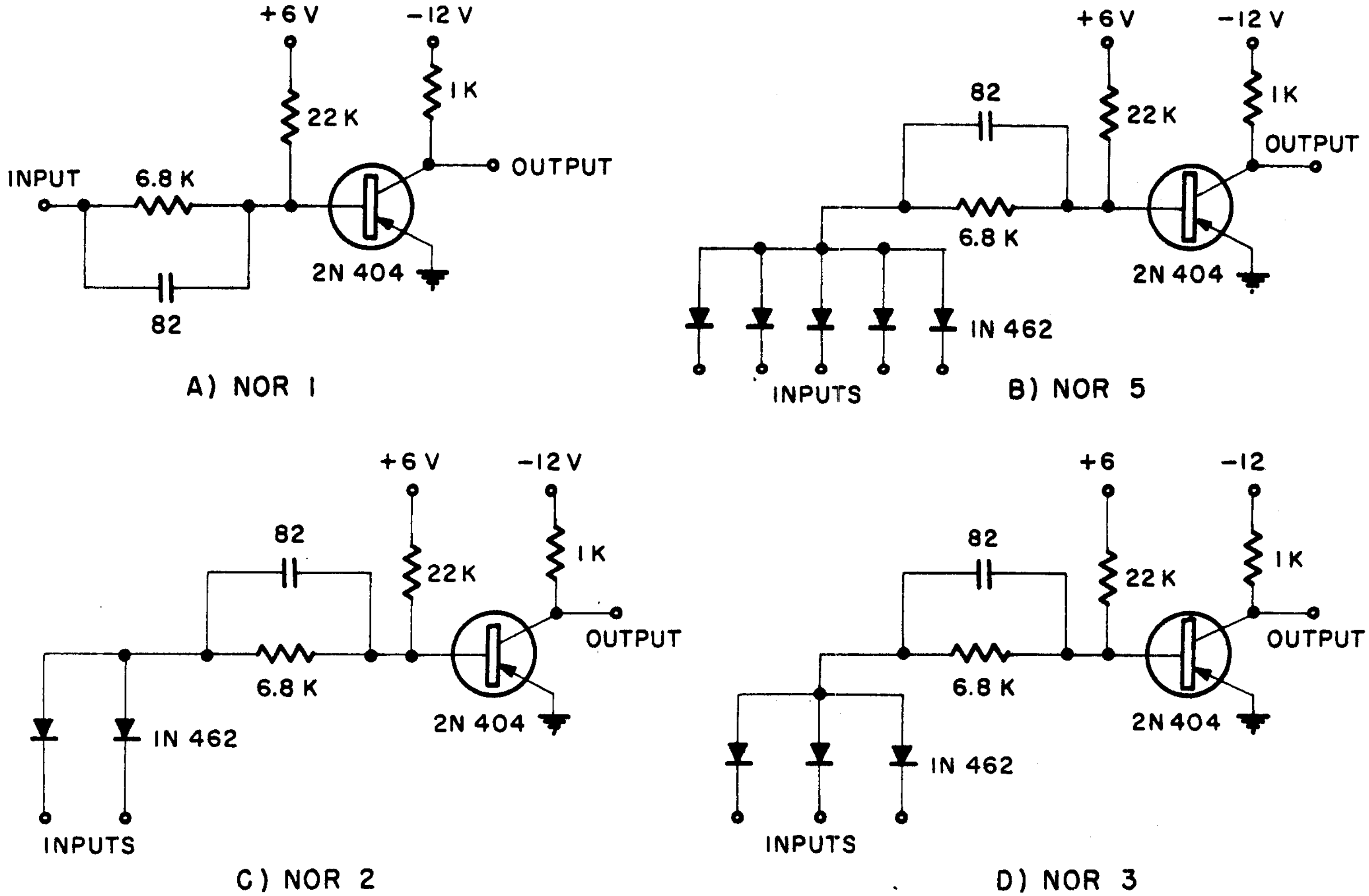


FIGURE 16 LOGIC MODULE CIRCUITS

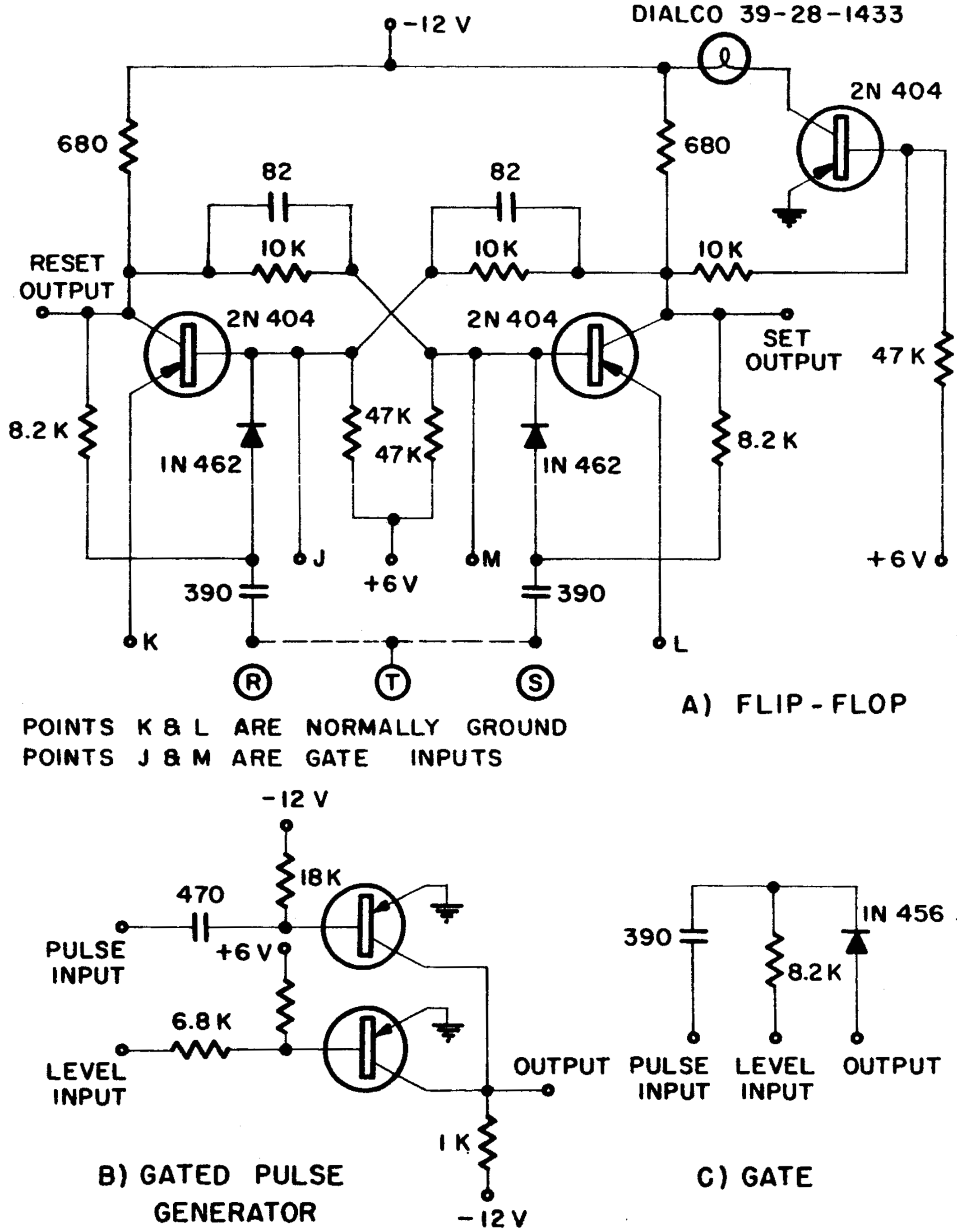


FIGURE 17 LOGIC MODULE CIRCUITS

the effective resolution as well as to determine the direction of rotation of the disc.

In Figure 18, a portion of such a disc is shown with the outputs properly spaced as indicated. Let the clear zone represent the true or asserted value of a Boolean function and the opaque zone represent the negated value. Also, let $\alpha = \bar{A} \rightarrow A$ transition and $\beta = A \rightarrow \bar{A}$ transition. When the shaft is rotated in the forward direction from the position indicated on the diagram, the following pulses will be produced successively: $A_2\alpha_1$, $A_1\beta_2$, $\bar{A}_2\beta_1$, $\bar{A}_1\alpha_2$ Similarly, if it is rotated in the backward direction, the following successive pulses will occur: $\bar{A}_2\beta_1$, $\bar{A}_1\alpha_2$, $A_2\alpha_1$, $A_1\beta_2$. Thus, the equations used to indicate the crossing of a quanta boundary and direction of crossing are

$$F = A_1\beta_2 + \bar{A}_1\alpha_2 + A_2\alpha_1 + \bar{A}_2\beta_1$$

$$B = A_1\alpha_2 + \bar{A}_1\beta_2 + A_2\beta_1 + \bar{A}_2\alpha_1$$

The outputs of the photo diodes are amplified by two transistor amplifiers mounted in the quantizer. The schematic of the output unit is shown in Figure 20. Ideally, the output waveform has sharp transitions between the voltage levels representing the clear and opaque zones; however, because of the finite width of the source and photodiode and

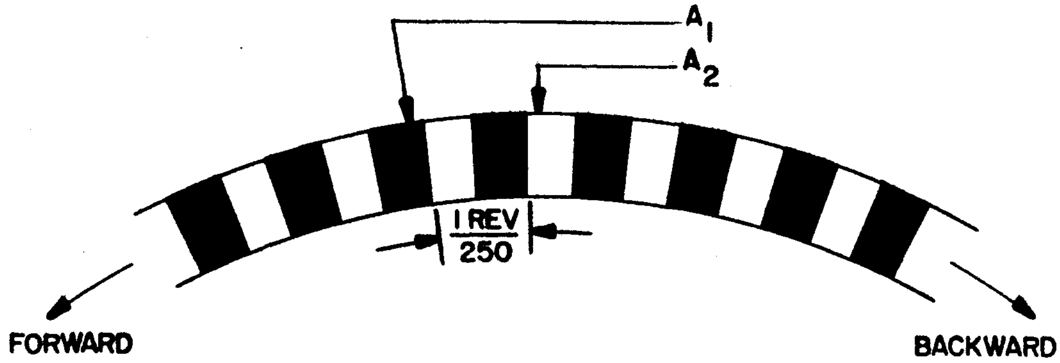


FIGURE 18 TYPICAL SEGMENT OF QUANTIZER DISC

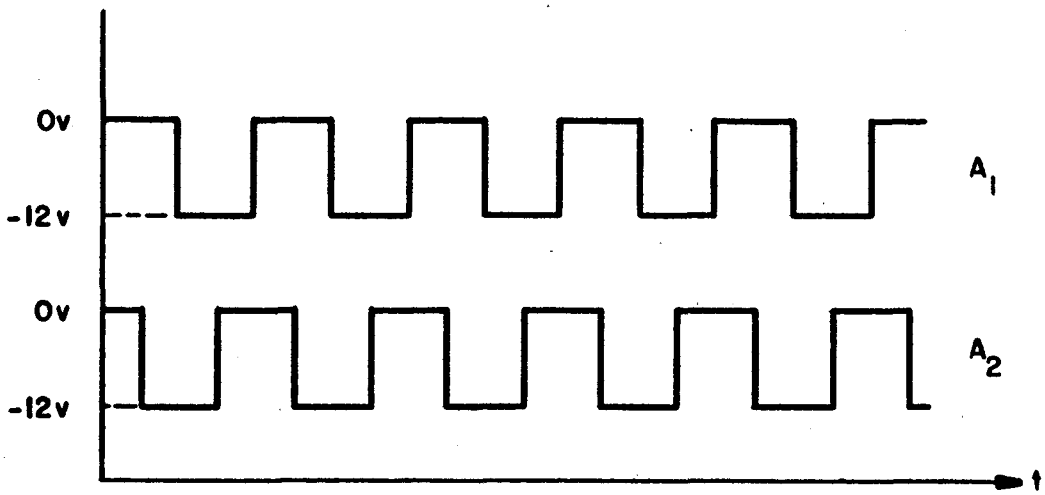
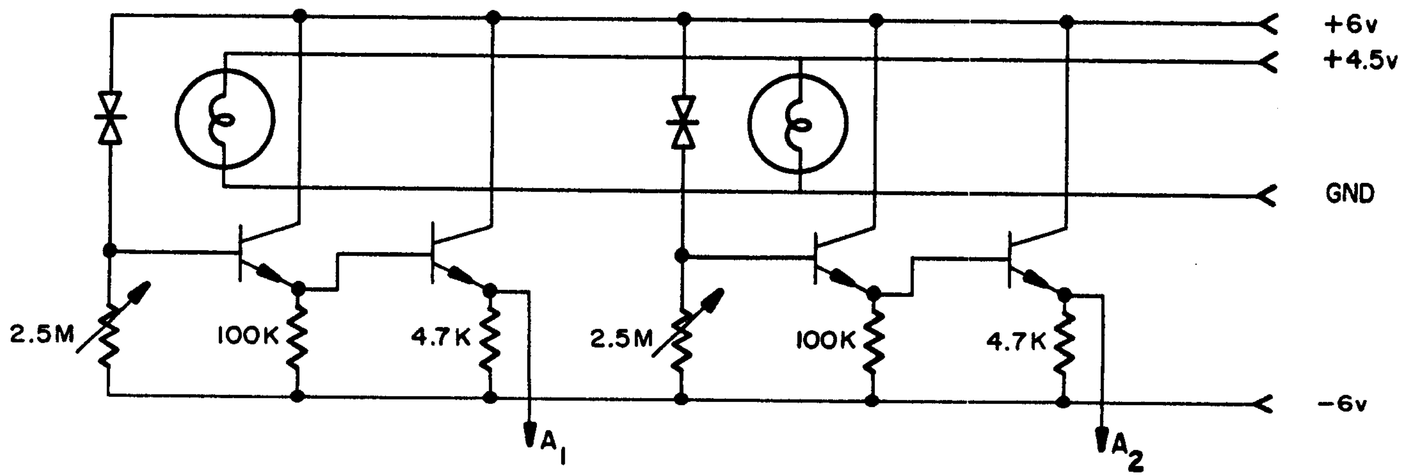


FIGURE 19 TYPICAL SHAPED OUTPUT OF SCHMITT TRIGGERS WITH QUANTIZER ROTATING AT CONSTANT SPEED IN FORWARD DIRECTION



ALL TRANSISTORS ARE TRS - 122
ALL PHOTODIODES ARE TIH - 11
ALL LAMPS ARE LOS ANGELES #7

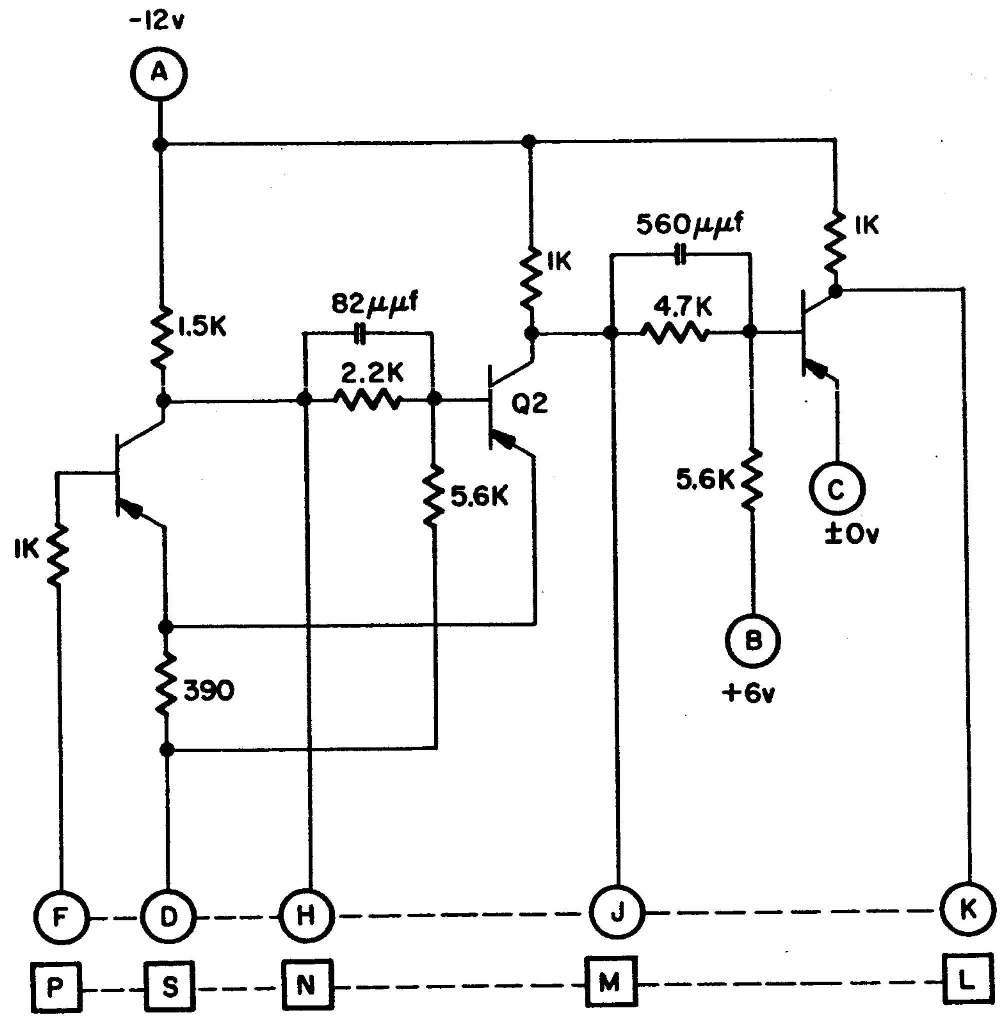
SOCKET CONNECTIONS

1	−6v	9	OUTPUT A_1
2		10	OUTPUT A_2
3	+6v	11	
4		12	
5		13	
6		14	
7	GND	15	CASE GND
8			

CANNON CONNECTOR DA-15P

FIGURE 20 WANG LABORATORIES #43 ENCODER CIRCUIT SCHEMATIC the lack of sharpness at the edge of the lines, the transition is not very well defined. A Schmitt trigger is inserted in each output line to provide a transition fast enough to be sensed by the direction determination logic. A Wang Schmitt Trigger Card No. 216A is used for this purpose as it is designed for use with the quantizer voltage levels and may be used in one of the external sockets of the Digital Synthesizer. A typical set of shaped output waves from the two Schmitt Trigger units with the quantizer moving in the forward direction at a constant speed is shown in Figure 19.

The squared output waves next enter a direction sensing unit which emits a pulse on one of two output lines indicating when a quanta boundary is crossed and the direction of rotation. A design is required which will implement the preceding equations. One such design is given in Figure 22; however, the circuit which is shown in Figure 23 has been designed by Wang Laboratories which performs the same functions in essentially the same manner, but achieves a slight reduction in the number of components and also allows the logic to be mounted on one standard size card which may be used in one of the external sockets of the synthesizer.



ALL TRANSISTORS 2N 404
TRIGGER LEVEL ≃ - 3 VOLTS WITH D OR S TIED TO ± 0v

FIGURE 21 TWO WANG TYPE 216 SCHMITT TRIGGERS

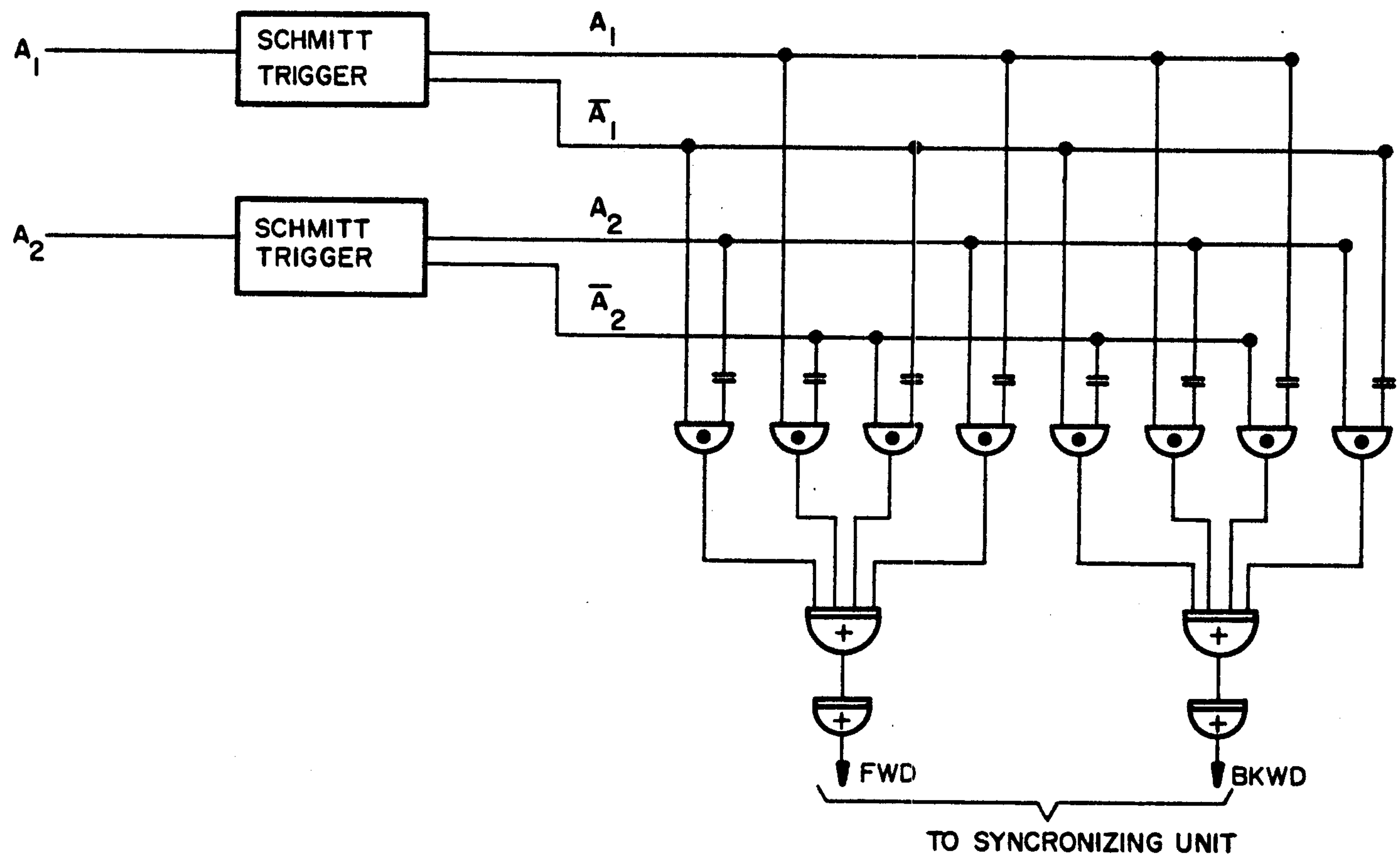


FIGURE 22 QUANTIZER DIRECTION SENSING UNIT

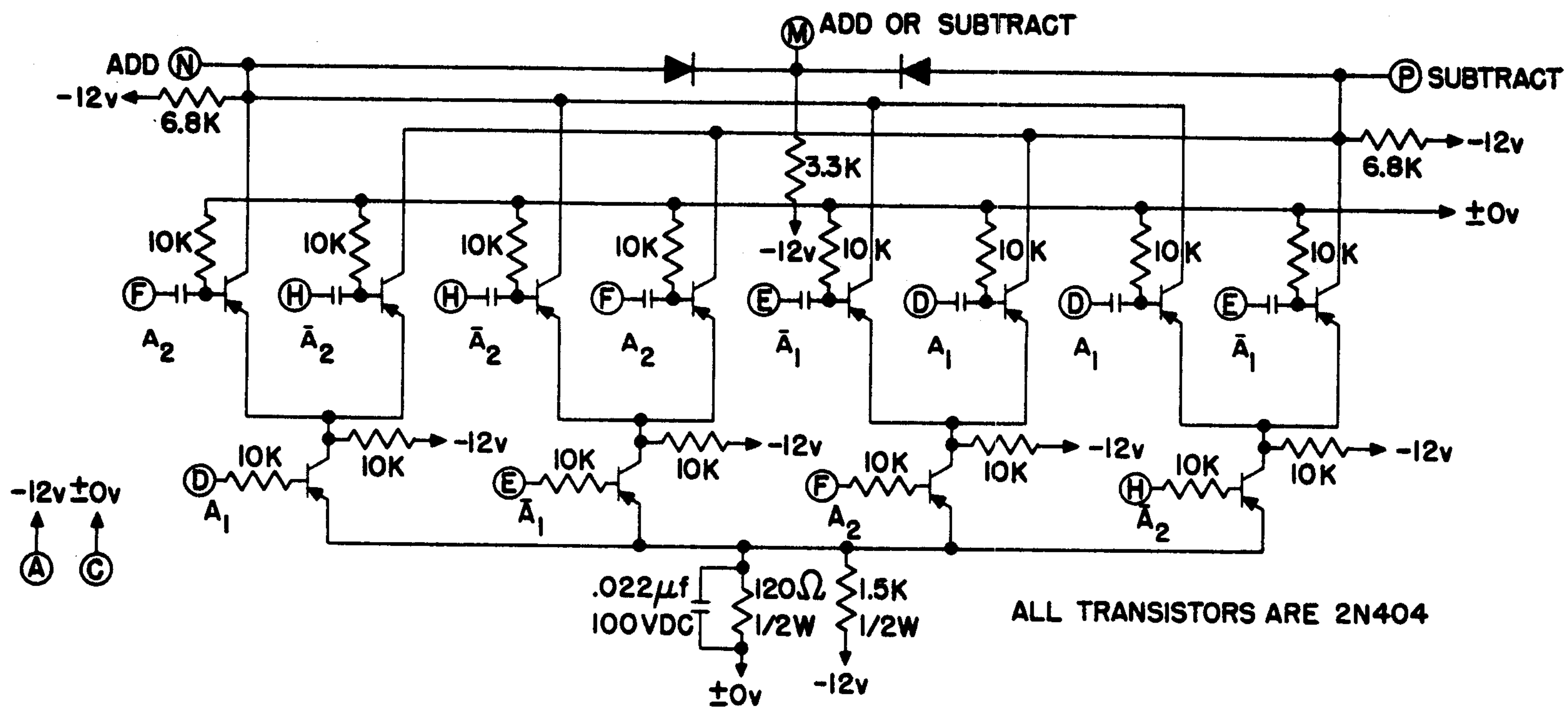


FIGURE 23 WANG LABORATORIES TYPE 242 DIRECTION SENSING SCHEMATIC

Synchronization and Timing

The purpose of this unit is to synchronize the input, feedback, and shift pulses with a basic clock frequency. These pulses are normally asynchronous with respect to one another and might otherwise enter the counters or shift registers in coincidence or near coincidence.

The design of the bidirectional counters requires that only one pulse at a time appear on the input lines; furthermore, the pulses must be separated by a time slightly greater than that required to complete the carry propagation. Failure to separate the pulses sufficiently will result in either a loss of a count or a complementation of the counter with quite unpredictable results. This further means that the synchronization frequency must be less than the maximum counter frequency.

It is also necessary to control the shift register input signals since no information may be transferred either into or out of the registers during the shift operation. The shift register also establishes a lower bound on the synchronization frequency as information will be lost if the synchronization frequency is less than the shift frequency.

It is thus necessary for the synchronization frequency to be greater than the maximum input, output, and shift frequencies but less than the maximum counter frequency; moreover, to reduce the sampling time, it is desirable to have the sync frequency as high as possible.

To achieve the pulse synchronization, it is necessary to provide a temporary storage or buffer element. A very simple circuit has been devised which utilizes a JK flip-flop and a gated pulse generator for each input line. The flip-flop acts as a buffer and is set each time the asynchronous pulse is applied. The synchronization pulse is applied to the reset side of the buffer element and if a change from the set to reset state is detected by the gated pulse generator, an output pulse is emitted. The requirement that the sync frequency is greater than the maximum input frequency means that there is no possible change for the asynchronous pulse to be lost. Note also that if the set and reset pulses occur at nearly the same time, the element will be set due to the JK characteristic and will be interrogated on the next pulse.

Position and Rate Counters

The following specifications are used in the design of the rate and position counters:

1. Counters must count in both forward and backward direction.

2. A standard binary code is to be used.

3. a. Position counter has 6 bits plus sign.
   b. Rate counter has 4 bits plus sign.

4. Input information
   a. Occurs in pulse form with lines normally at ground potential.

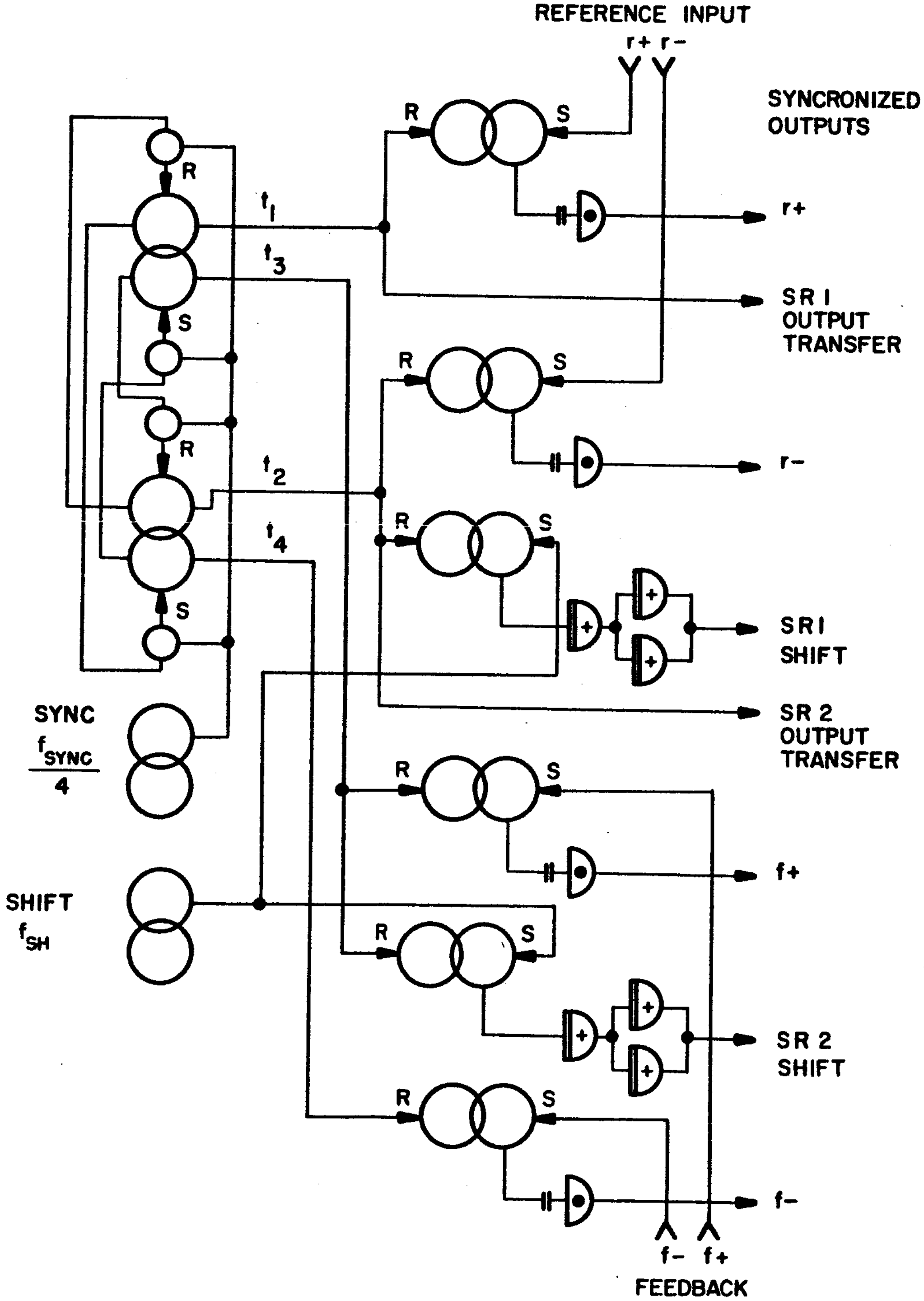


FIGURE 24 SYNCRONIZATION AND TIMING LOGIC

b. Pulses are separated by a time greater than the count time.

5. The output information will provide both sign and magnitude information in standard binary form.

Three methods have been considered to achieve a design which meets these requirements. The first method uses a conventional bidirectional counter with the n^{th} bit used as a zero bias point. In this case the sign is represented by the state of the n^{th} bit and the other bits indicate the magnitude. Negative numbers, however, appear in a 2's complement form and to avoid a rather complicated gating arrangement, it is necessary to read the 1's complement. This means that a "double zero" point occurs when making transitions through the zero bias point.

The second counter considered is shown in Figure 25. This uses a conventional bidirectional counter with an additional bit used for sign indication. The magnitude of the counter is increased if the sign is positive and a positive count pulse is received or if the sign is negative and a negative count pulse is received. In the other two cases, the magnitude is decreased. The sign bit, which controls the direction of magnitude count, is modified only when the counter makes a transition through zero causing an underflow in the magnitude flip-flops--this transition is used to trigger the sign bit and correct the magnitude to a value of 00. . . 01. These two cases are shown on the next page.

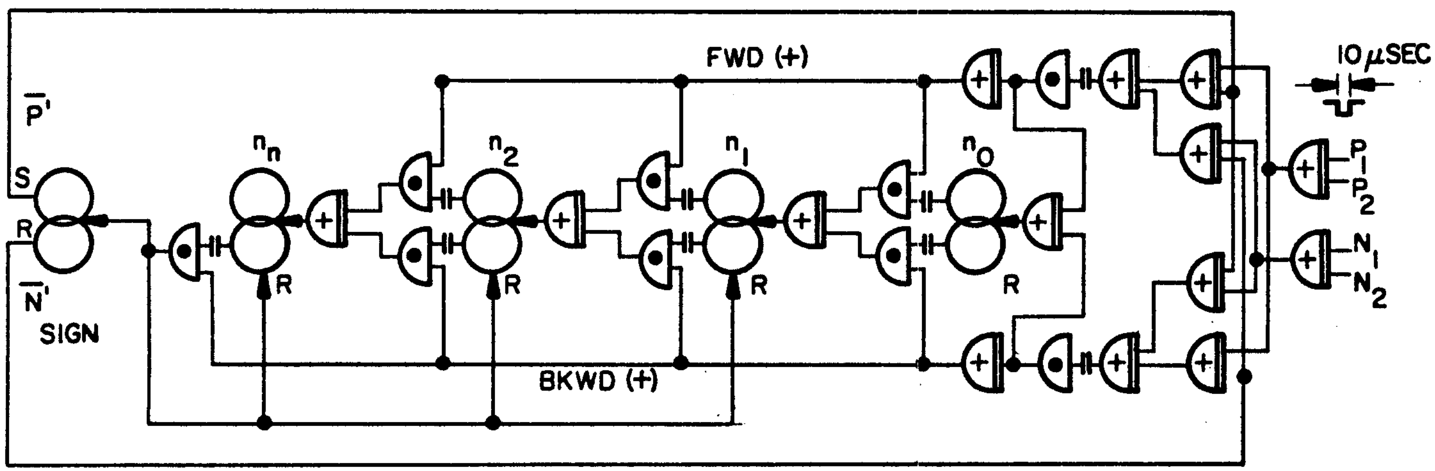


FIGURE 25 4-BIT BIDIRECTIONAL MAGNITUDE & SIGN BINARY COUNTER

- 1	1 0 0 0 1	+ 1	0 0 0 0 1
(-) 0	1 0 0 0 0	(+) 0	0 0 0 0 0
	(0 1 1 1 1)		(1 1 1 1 1)
+ 1	0 0 0 0 1	-1	1 0 0 0 1

Transition States

Letting

P = positive count input pulse

N = negative count input pulse

P' = positive sign in counter

N' = negative sign in counter

the counter magnitude direction is

$$FWD = (P_1 + P_2)P' + (N_1 + N_2)N'$$

$$BKWD = (P_1 + P_2)N' + (N_1 + N_2)P'$$

The equations for the trigger inputs of the flip-flops are

$$T'_o = (P_1 + P_2 + N_1 + N_2) = FWD + BKWD$$

$$T'_K = (FWD)\beta_{k-1} + (BKWD)\alpha_{k-1}$$

where $n \geq k \geq 1$. The equations for the reset inputs of the flip-flops for count correction due to a transition through zero are

$$R'_K = (BKWD)\alpha_1$$

$$T'_{sgn} = (BKWD)\alpha_n$$

Caution must be taken to insure that the zero transition corrections are not initiated until the "direction enable" lines inhibit the propagation of carry pulses between stages during the correction process.

The necessity of count correction in this case reduces the maximum operation frequency and also reduces the operation reliability. When this circuit was tested using the synthesizer, it performed moderately well; however, because of noise and crosstalk problems in the synthesizer which have only recently been reduced, its operation was not acceptable for use in the actual system.

As a result, a third design was chosen for use which uses more logic than the above method, but appears to be much more reliable. In this counter, an actual zero detection network is used and whenever the count is zero, the sign is set to that of the input count pulse and the magnitude is counted in the forward direction. The rest of the count operation is the same as that in the above counter. Thus,

$$\mathrm{FWD} = (\mathrm{ZERO})\,(P_1 + P_2 + N_1 + N_2) + (P_1 + P_2)\,P' + (N_1 + N_2)\,N'$$

$$\mathrm{BKWD} = (\overline{\mathrm{ZERO}})\;[(P_1 + P_2)\,N' + (N_1 + N_2)\,P']$$

$$S'_{sgn} = (\mathrm{ZERO})\;(P_1 + P_2)$$

$$R'_{sgn} = (ZERO)\,(N_1 + N_2)$$

$$T'_o = FWD + BKWD$$

$$T'_K = (FWD)\,\beta_{k-1} + (BKWD)\,\alpha_{k-1}$$

To improve the counter frequency, the leading edge of the carry pulses is used to trigger the flip-flops. The logical diagram for this counter is shown in Figure 26.

Shift Registers

The function of the two shift registers is to delay the input pulse train for a selected period of time. The delay time will be approximately

$$\tau = \frac{n_{sh}}{f_{sh}}$$

where

n_{sh} = number of bits in the shift register

f_{sh} = shift frequency.

The design of the shift register is based upon the analysis given below. When a shift pulse occurs, the information in the shift register is to be simultaneously transferred to the next memory element in the register. As a result, the value of the k^{th} bit after the shift pulse is to be that of the k-1 bit before the shift pulse or, using an RS type memory element,

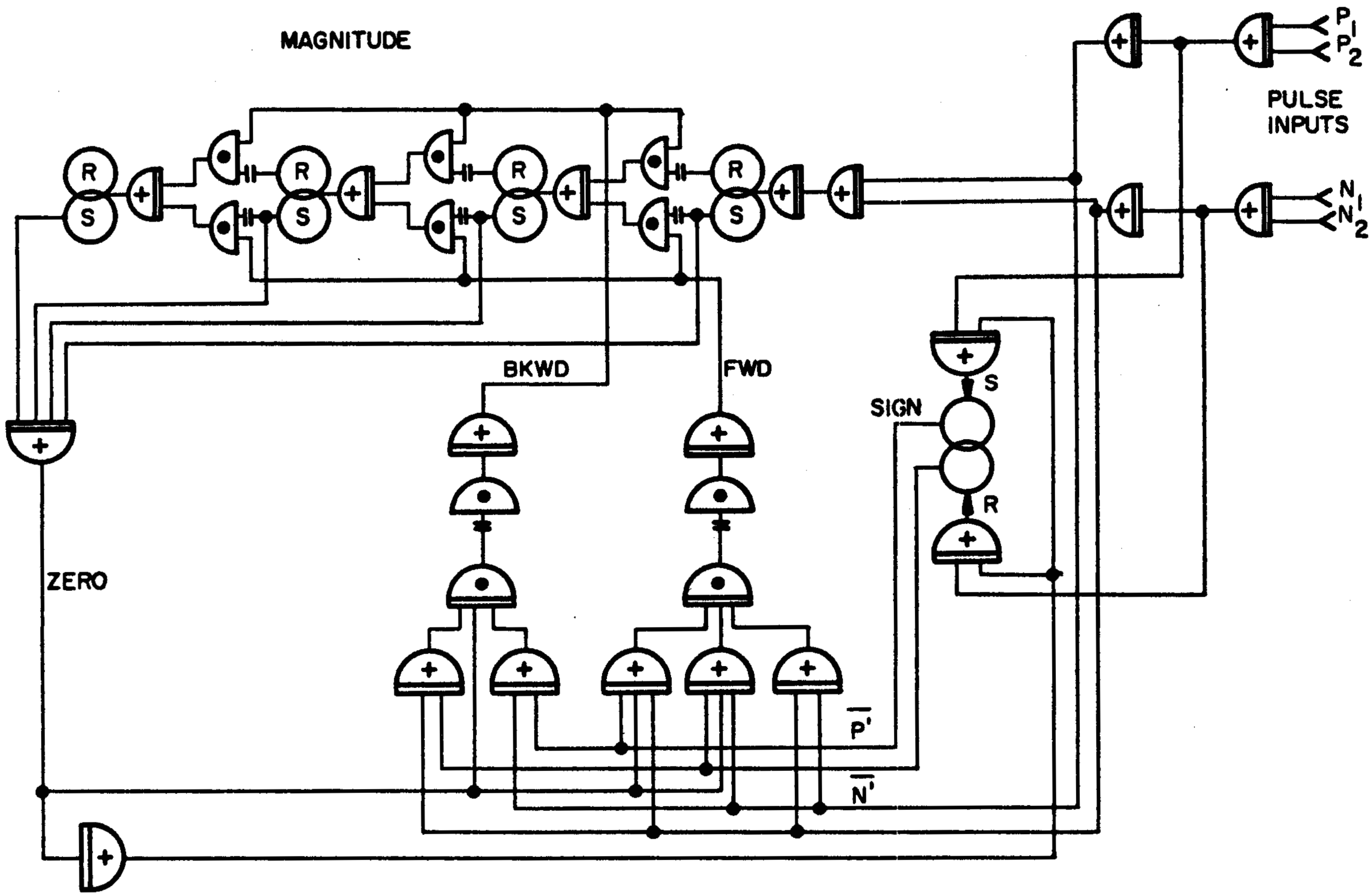


FIGURE 26 4-BIT SIGN & MAGNITUDE BIDIRECTIONAL BINARY COUNTER these conditions are

$$R'_K = (R'_{k-1})(P_{sh}) = (\overline{S'_{k-1}})(P_{sh})$$

$$S'_K = (S'_{k-1})(P_{sh}) = (R'_{k-1})(P_{sh})$$

where $2 \leq k \leq n$. Both the Wang steering gate and the gated pulse generator accept level and pulse information in exactly the form shown in the second parts of the above equations, and since the steering gates use passive elements and are designed for use into the flip-flop elements, they are used to implement these equations as shown in Figure 27.

The disadvantage of the above shift technique is that the output may not be interrogated during the shift operation because of possible transitions in the n^{th} bit; moreover, it is also necessary to properly synchronize the input pulse to be out of phase with the shift pulse. The information output is controlled by using the last flip-flop as a buffer element which is interrogated by the synchronization unit, and the input and output information transfer pulses are separated from the shift pulses.

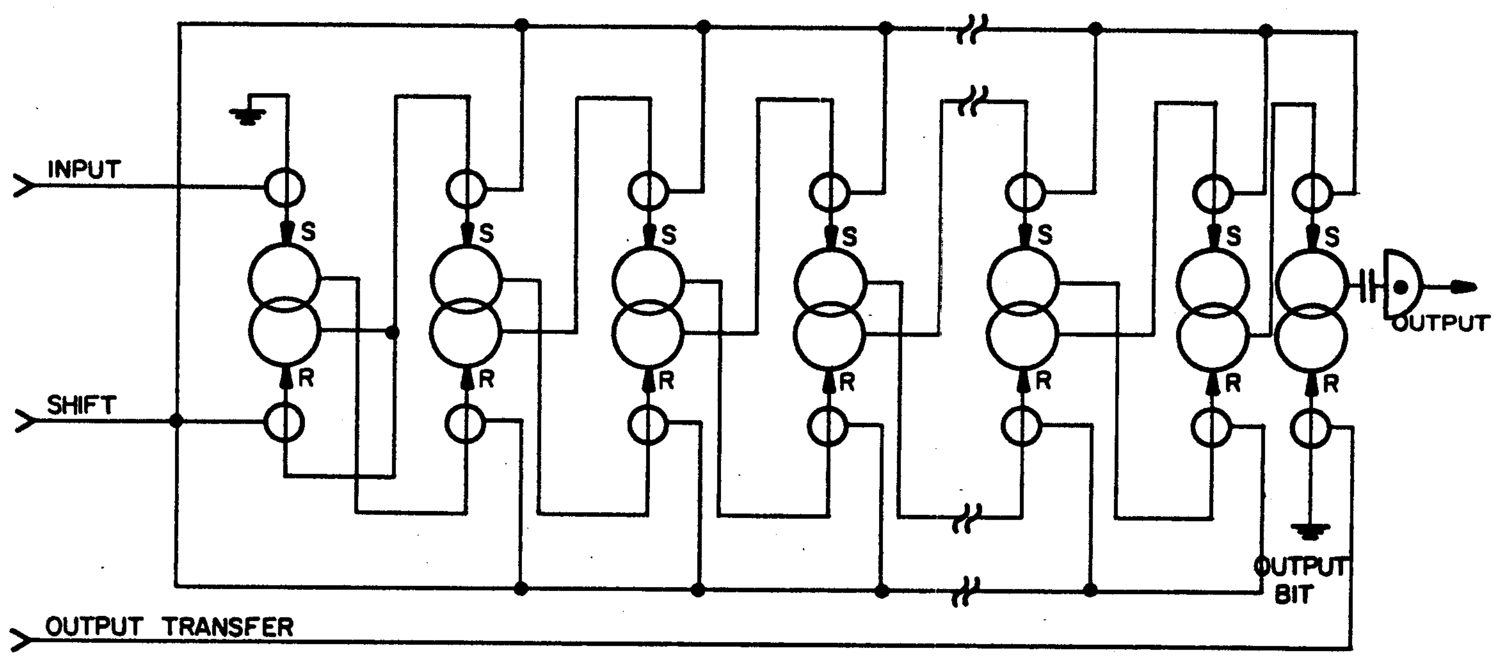


FIGURE 27 . SHIFT REGISTER

Digital to Analog Converter

A voltage ladder network is used to perform the conversion from the digital magnitude indicated by the bidirectional counter to an equivalent analog voltage.

In the circuit shown in Figure 28, it is assumed that the impedance of the voltage sources is negligible compared with the value of R and the load resistance is infinite with respect to R. Since there are no active elements in the circuit, the principle of superposition may be applied and the output voltage will be the sum of the voltage due to each source considered independently with the others shorted.

The output voltage due to e_n, considering the network as a voltage divider (Figure 29a), is

$$E_{o,n} = e_n \left(\frac{2R}{R + 2R}\right) = e_n \left(\frac{1}{2}\right)$$

The voltage due to e_{n-1} is (Figure 29b)

$$E_{o,n-1} = e_{n-1} \left(\frac{2R}{R + 2R}\right) \frac{\frac{6}{5} R}{\frac{6}{5} R + 2R} = e_{n-1} \left(\frac{1}{4}\right)$$

A similar analysis shows that for any source, k, the voltage will be

$$E_{o,k} = e_k \frac{1}{2^{1+n-k}}$$

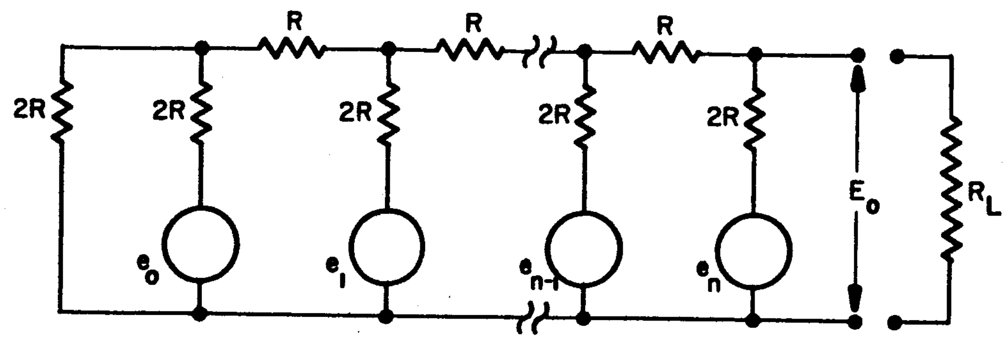
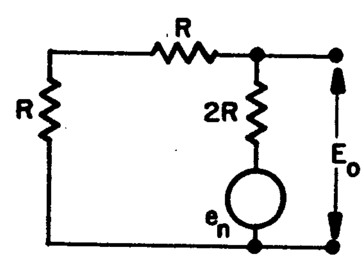
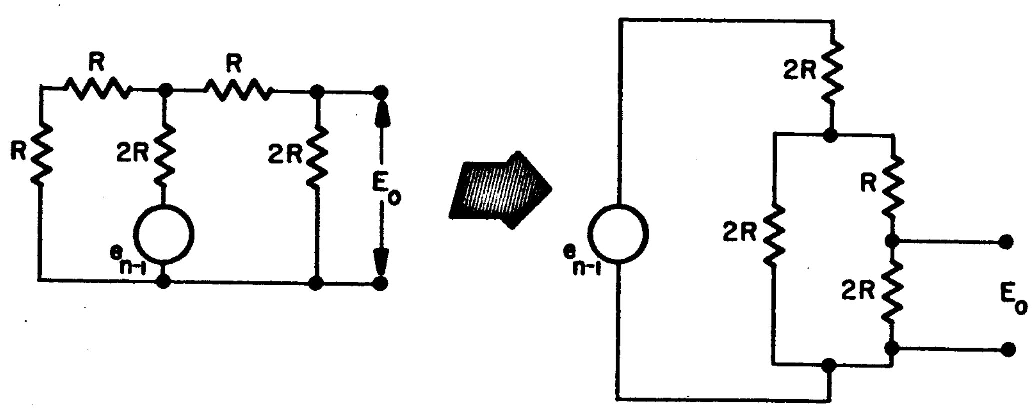


FIGURE 28 DIGITAL TO ANALOG CONVERTER
VOLTAGE LADDER



(a)



(b)

FIGURE 29 DETERMINATION OF OUTPUT VOLTAGE
(a) DUE TO e_n
(b) DUE TO e_{n-1}

The total output voltage will then be

$$E_o = e_o \left(\frac{1}{2^{n+1}}\right) + e_1 \left(\frac{1}{2^n}\right) + \ldots + e_n \left(\frac{1}{2}\right)$$

$$E_o = \left(\frac{1}{2^{n+1}}\right) \left(e_o + e_1 2^1 + \ldots + e_n 2^n\right)$$

The voltage sources, e_o, e_1, ... e_n, will be allowed to have only two values--E and ground, and a binary number, D,, will be used to describe the value; therefore,

$$D = e_1 2^1 + e_2 2^2 + \ldots + e_n 2^n$$

where all e_k = 0 volts or E volts. As a result, the output voltage will be

$$E_o = \left(\frac{E}{2^{n+1}}\right) \left(d_o 2^o + d_1 2^1 + \ldots + d_n 2^n\right.$$

$$E_o = \left(\frac{E}{2^{n+1}}\right) (D)$$

In actual practice, if the load resistance is not infinite, it will act as another voltage divider and reduce the output voltage. A set of Wang D/A converter cards no. 215 was selected to be used in the system, but the cards were modified slightly as shown in Figure 30, to allow use in the synthesizer because of the poser connections on the first three pins. The actual performance was tested and the output appeared to be quite linear with -6v used as the source voltage.

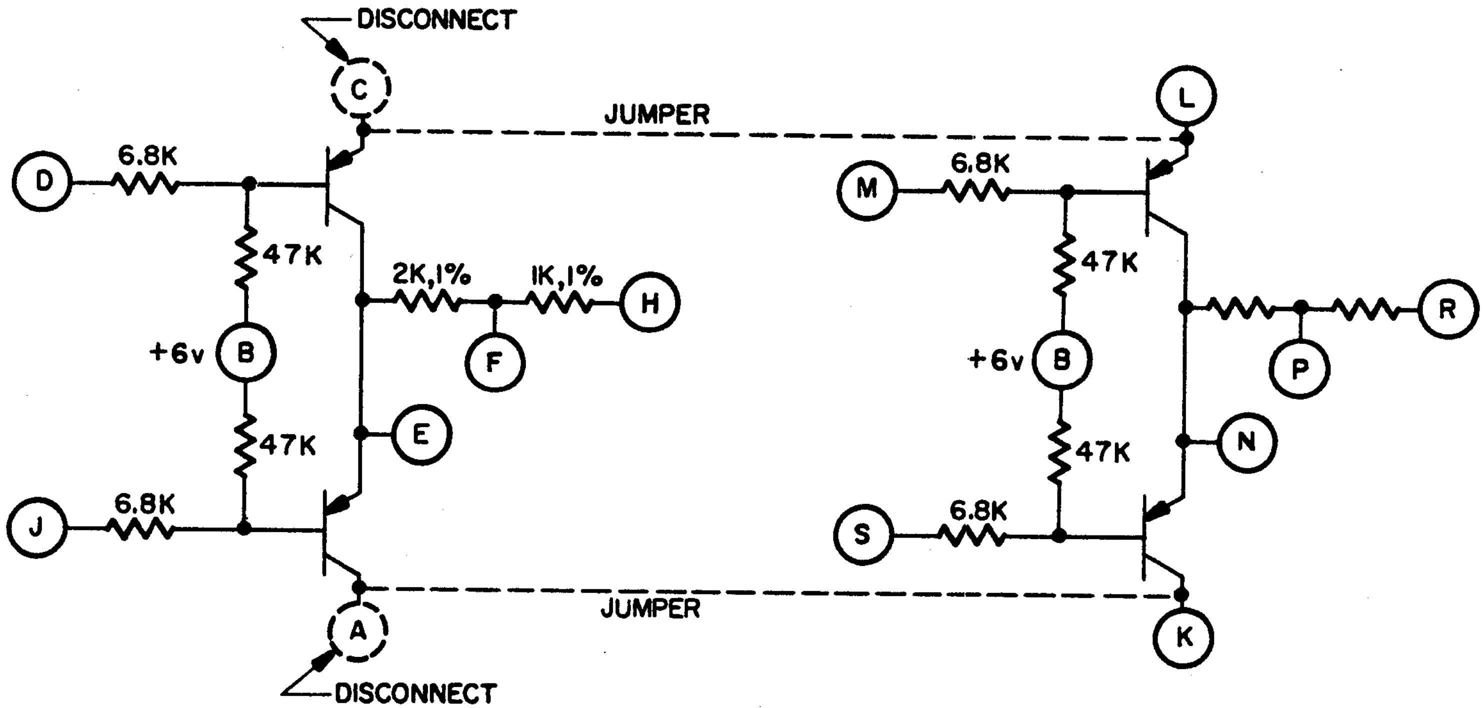


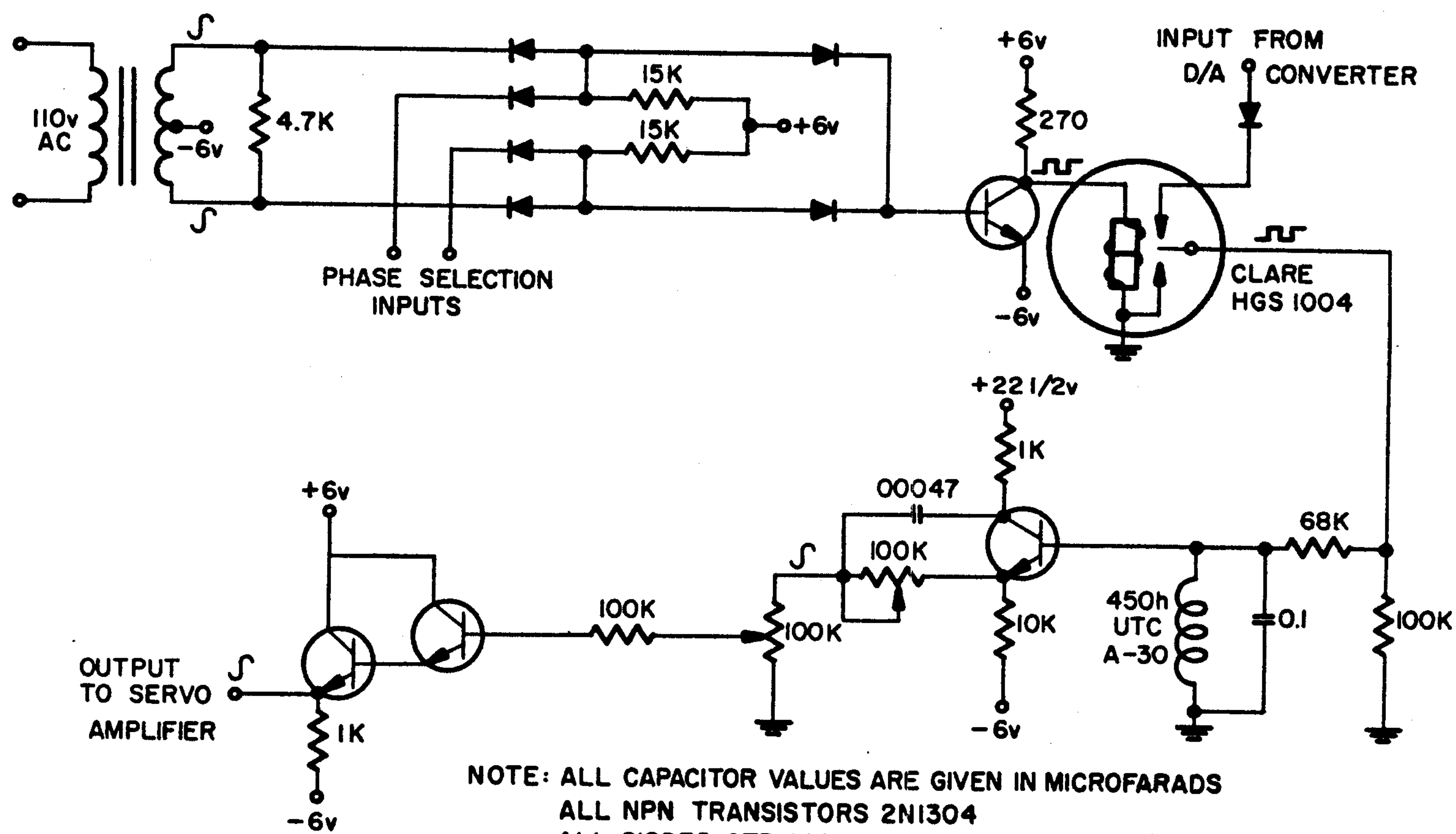
FIGURE 30 WANG LABORATORIES #215
DIGITAL TO ANALOG CONVERTERS

Phase Controlled Modulator

The purpose of this unit is to produce a 60 cps output signal with the magnitude proportional to the magnitude of the associated bidirectional counter and the sign is used to produce a phase lead or lag of 90° with respect to the servomotor reference signal. The actual circuit used was designed by Marion Kosem[1] and is shown in Figure 31.

In this unit, the reference voltage is applied to the primary of a 6.3 volt filament transformer. The secondary is center tapped so that the voltages between this tap and the ends are 180° out of phase. The center tap is biased at -6v to allow the connection of the ends of the secondary to one leg of two 2 input "AND" gates. The two outputs of the sign bit of the counter are connected to the second inputs and the outputs of the two gates are each connected to an input of an "OR" gate. Thus the sign bit can select the a-c signal with the desired phase.

This a-c signal is then used to drive a Clare Mercury-Wetted Relay through a chopper amplifier. The analog output of the D/A converter is applied to one contact of the relay and the other contact is grounded. The output line will then consist of an alternate ground and voltage with the desired phase. This signal is passed through an LRC filter to change the waveshape to a sinusoidal pattern and then through a minor phase shift



NOTE: ALL CAPACITOR VALUES ARE GIVEN IN MICROFARADS
ALL NPN TRANSISTORS 2N1304
ALL DIODES CTP 462

FIGURE 31 PHASE CONTROLLED MODULATOR

network to adjust the output phase so that the angle between this and the reference phase will have the correct value.

Servo Amplifier and Motor

The motor used is a Dhiel Model FPE 49-56-2 low inertia two phase a-c servomotor with a high impedance control winding and a tachometer winding. The magnitude of the a-c signal which is applied to the control windings determines the speed of the motor and the phase of the signal with respect to the reference signal determines the direction of rotation.

The amplifier used to control the motor was developed by C. C. Crabs and is shown in Figure 32. Note that the high impedance and center tap of the servomotor winding allows the push-pull output to be applied directly to the motor winding.[4]

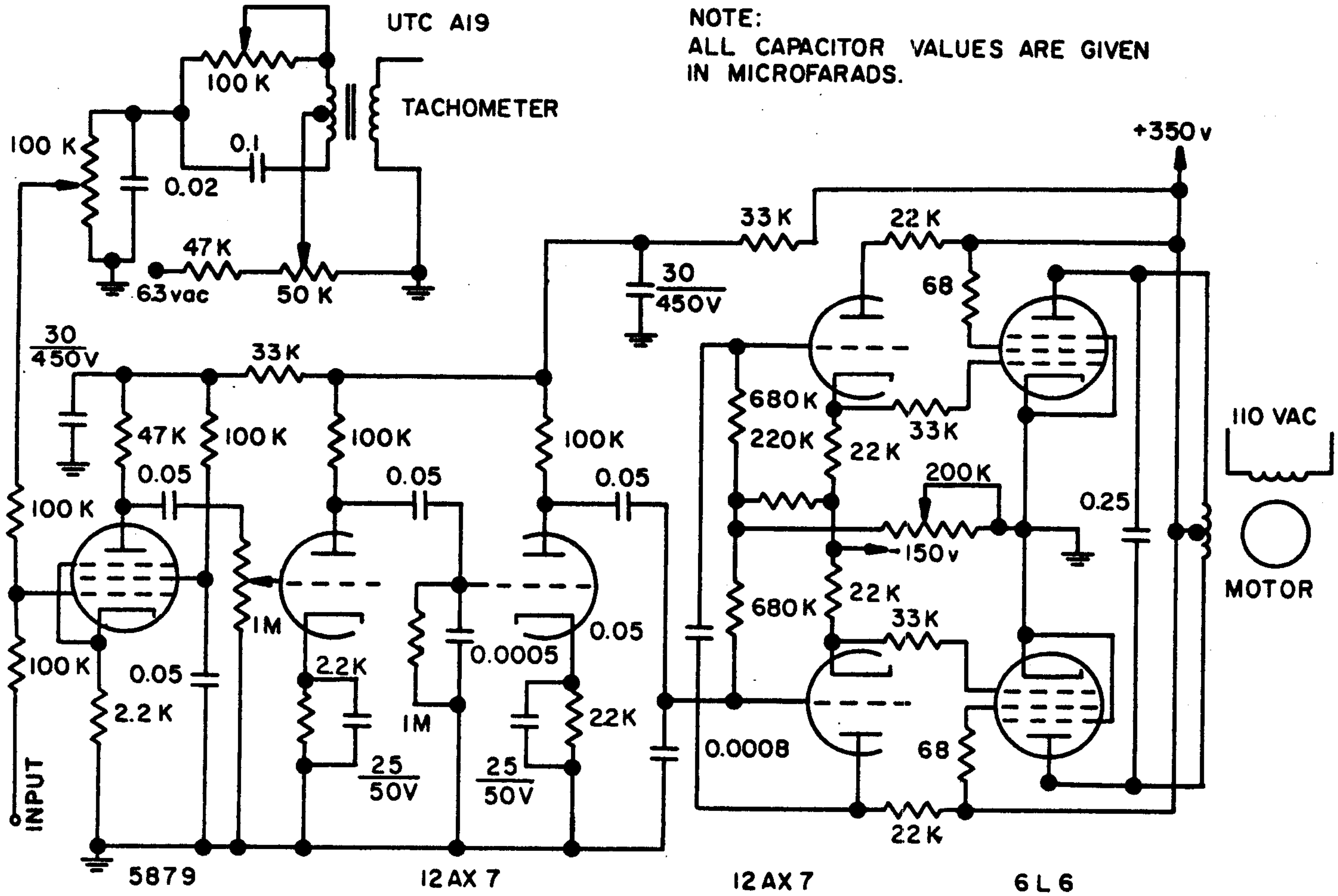


FIGURE 32 SERVO AMPLIFIER

APPENDIX II
COMPUTER PROGRAM DESCRIPTION

Introduction

The computer program shown in Figure 33 was written to compute the output position (in number of quantum) as a function of the reference input and the parameters K_P, K_T, T_L and τ using equations (18) and (23). As the times at which the feedback pulses occur are a function of $\theta_o(t)$ and are initially known, it is necessary to determine $\theta_o(t)$ by assuming an initial value of $\theta_o(o) = 0$. The output is then determined for a time slightly greater than zero and the process is continued for a specified amount of time. Each time the value of $\theta_o(t)$ passes through an integer value, a new feedback pulse is generated and the sign indicates the direction of the transition. A brief summary of this process is outlined below:

1) Initialize $\theta(t) = 0$ at $t = 0$, all feedback pulses = 0.
2) Read values of K_T, K_P, T_L, τ, A_n, T_{rn}.
3) Increment t by Δt.
4) Compute value of $\theta_o(t)$.

```
COMMENT INCREMENTAL DIGITAL SERVOSYSTEM WITH DIGITAL RATE FEEDBACK$
INTEGER M,N,MMAX,NMAX,X,QUANT,BIAS,A(),B()  $
ARRAY A(100), TREF(100), B(1000) , TEBK(1000)  $
FUNCTION Q(T) = T-0.125+(0.125)(EXP (-T/0.125)) $
LBL1.. FOR N=(1,1,100)$BEGIN A(N)=0.0$TREF(N)=0.0 END$READ ($$REF)$
       FOR N=(1,1,NMAX)$ WRITE ($$RERN,FORM3)$
LBL2.. FOR M=(1,1,1000)$BEGIN B(M)=0.0$TEBK(M)=0.0 END$
       READ ($$DATA)$WRITE ($$DAT,FORM1)$WRITE ($$HEAD)$
       BIAS = 10 $ EPS = 0.002 $
       QUANT = BIAS $ T = DTMIN $ MMAX = 0 $ TLAST = 0.0 $
LBL3.. THLST=THETA$THETA=0.0$
LBL4.. FOR N=(1,1,NMAX)$BEGIN IF T LSS TREF(N)$GO TO LBL5$
       THETA=THETA+(A(N))(KP)(Q(T-TREF(N)))$END$
LBL5.. IF MMAX EQL 0  $  GO TO LBL7  $ FOR M = (1,1,MMAX) $ BEGIN
       TE = T - TEBK(M) $ TED = TE - DELAY $ IF TED LSS 0.0 $ TED=0.0  $
       THETA=THETA+(B(M))((KT)(Q(TED))-(KT+KP)(Q(TE)))$END$
LBL7.. IF B(MMAX+1) EQL 0$BEGIN X=FIX (THETA) +BIAS-QUANT$
       IF X EQL 0$GO TO LBL9$IF ABS (X) GTR 1$GO TO ER1$
       THHGH=THETA$THLOW=THLST$THGH=T$TLOW=TLAST$B(MMAX+1)=X$END$
LBL8.. TEMP=THETA-(QUANT-BIAS+(1+X)/2)$IF ABS (TEMP) GTR EPS $ BEGIN
       TLAST=T$T=T-(THGH-TLOW)(TEMP)/(THHGH-THLOW)$
       IF FIX (THETA) +BIAS-QUANT EQL 0$BEGIN
       THLOW=THETA$TLOW=TLAST$GO TO LBL3$END$
       THHGH=THETA$THGH=TLAST$GO TO LBL3$END$
       MMAX=MMAX+1$IF MMAX GEQ 1000$GO TO ER2$TEBK(MMAX)=T$
       QUANT = QUANT + X     $
       WRITE ($$ANS,FORM2)$
LBL9.. TLAST=T$T=T+DELT $
LBL10.. IF T GTR TMAX$GO TO LBL2$GO TO LBL3$
ER1.. THOLD=T$T=T-ABS ((T-TLAST)/2.0)$TLAST=THOLD$GO TO LBL3$
ER2.. WRITE ($$OVER) $ GO TO LBL2 $
INPUT DATA (KP,KT,DELT ,DELAY,TMAX)$
INPUT REF (NMAX, FOR N=(1,1,NMAX)$(A(N),TREF(N))) $
OUTPUT DAT (KP,KT,DELAY)$
OUTPUT ANS (T, THETA )     $
OUTPUT RERN (N,TREF(N),A(N))$
FORMAT FORM1 (*KP=*,X6.2,B2,*KT=*,X6.2,B2,*DELAY=*,X8.6,W3)$
FORMAT FORM2 (B 3, X20.5 , B20, X20.5 , W0)   $
FORMAT FORM3 (*N=*,I4,B5,*TREF=*,X6.2,B5,*A(N)=*,I4,W4)$
FORMAT HEAD (B20, *T*, B35, *THETA*, W6)  $
FORMAT OVER (*ALLOTED CAPACITY EXCEEDED*, W0) $
    FINISH  $
```

FIGURE 33 COMPUTER PROGRAM

5) If $\theta_o(t)$ passes through an integer value
   a) Set $B_m = \pm 1$, sign determined by direction;
   b) Use interpolation technique to determine time of crossing;
   c) When time of crossing is determined, set $T_{fm} = t$.
6) Return to step 3 and repeat computations until $t \geq T_{max}$.

The flow diagram, Figure 34, indicates the detailed information flow which was used to formulate the program. Several of the important details and points of interest are described in the sections below.

Determination of Quanta Boundary Transition

In order to detect an increase of + 1 or - 1 from the previous quanta boundary or a repeated crossing of the last boundary, a feature of the computer is used which allows the truncation of a real number to its integer value. In the operation, the value of $\theta_o(t)$ is truncated to its corresponding integer value, and the integer value of the previous quanta boundary is subtracted. This difference simply represents the number of quanta boundaries crossed and the sign represents the direction of rotation. If the magnitude is two or greater, the value of t must be reduced, as it is necessary to determine the time of crossing of every quanta **boundary**.

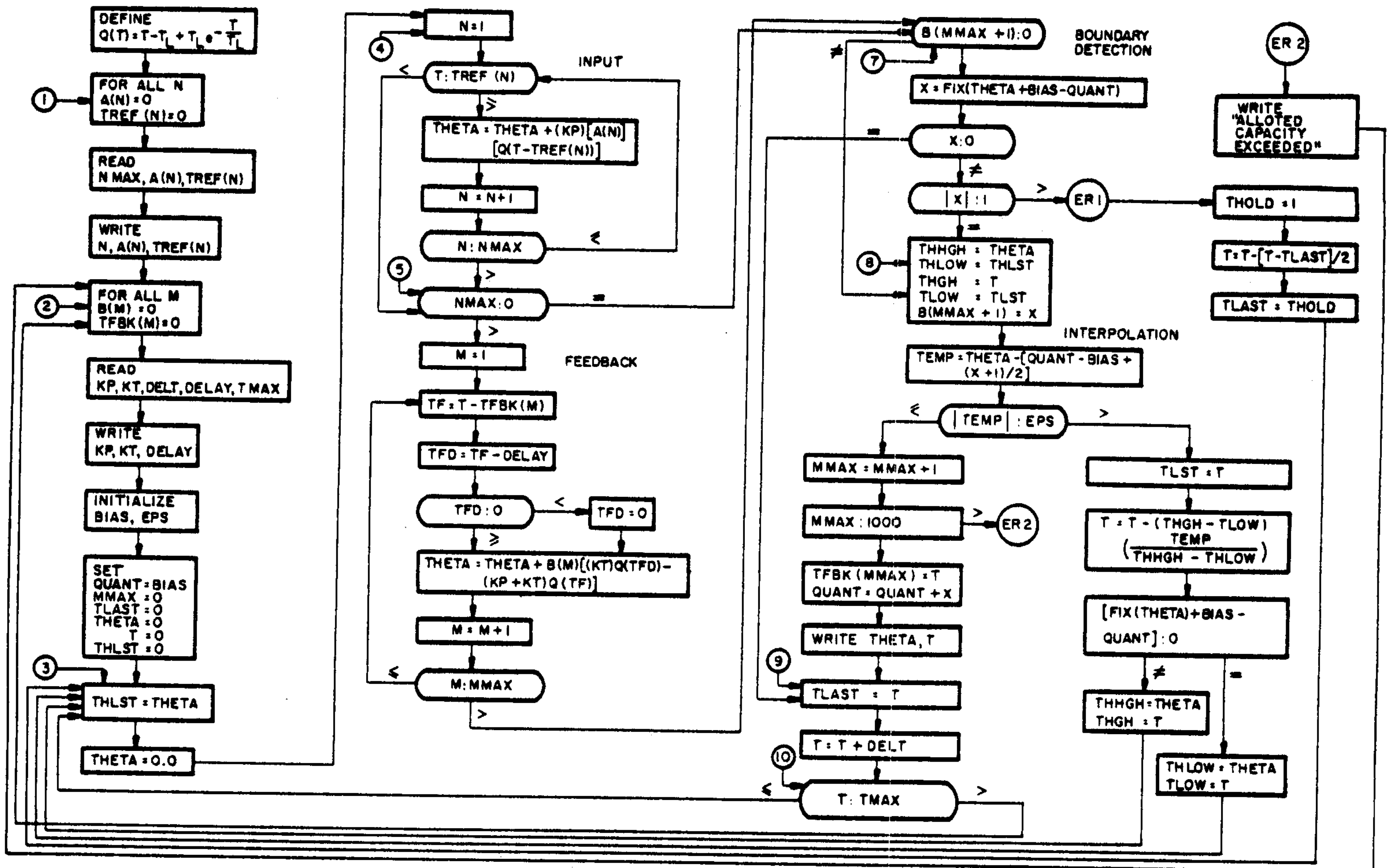


FIGURE 34 COMPUTER FLOW DIAGRAM

Bias

Note that in the preceding discussion, it is necessary for the value of the quanta boundary to always be positive; otherwise, the crossing of $\theta_o(t) = 0$ will not be detected. To insure this, a BIAS parameter is included in the program. It will be necessary to make this value high enough to insure that the value of $\theta_o(t)$ + BIAS remains positive without exceeding the upper magnitude limit (specified by the computer word length).

Time Determination of Quanta Boundary Transition

Whenever it is determined that a quanta boundary has been crossed, it is necessary to determine the time at which this event occurred. This is accomplished by using the values of $\theta_o(t)$ just before and just after the transition to interpolate (linearly), and using the resulting value of t, the output is computed. The process is continued until the value of $\theta_o(t)$ is within a value ϵ from the boundary. An example of the four possible cases is shown in Figure 35. The parameters θ_{HI} and θ_{LO} are used to indicate the value of $\theta_o(t)$ just before and just after the quanta boundary respectively; therefore, in the interpolation scheme, either θ_{LO} or θ_{HI} is modified dependent upon the value of the output compared with the quanta boundary and the direction of the transition. In this manner, the distance

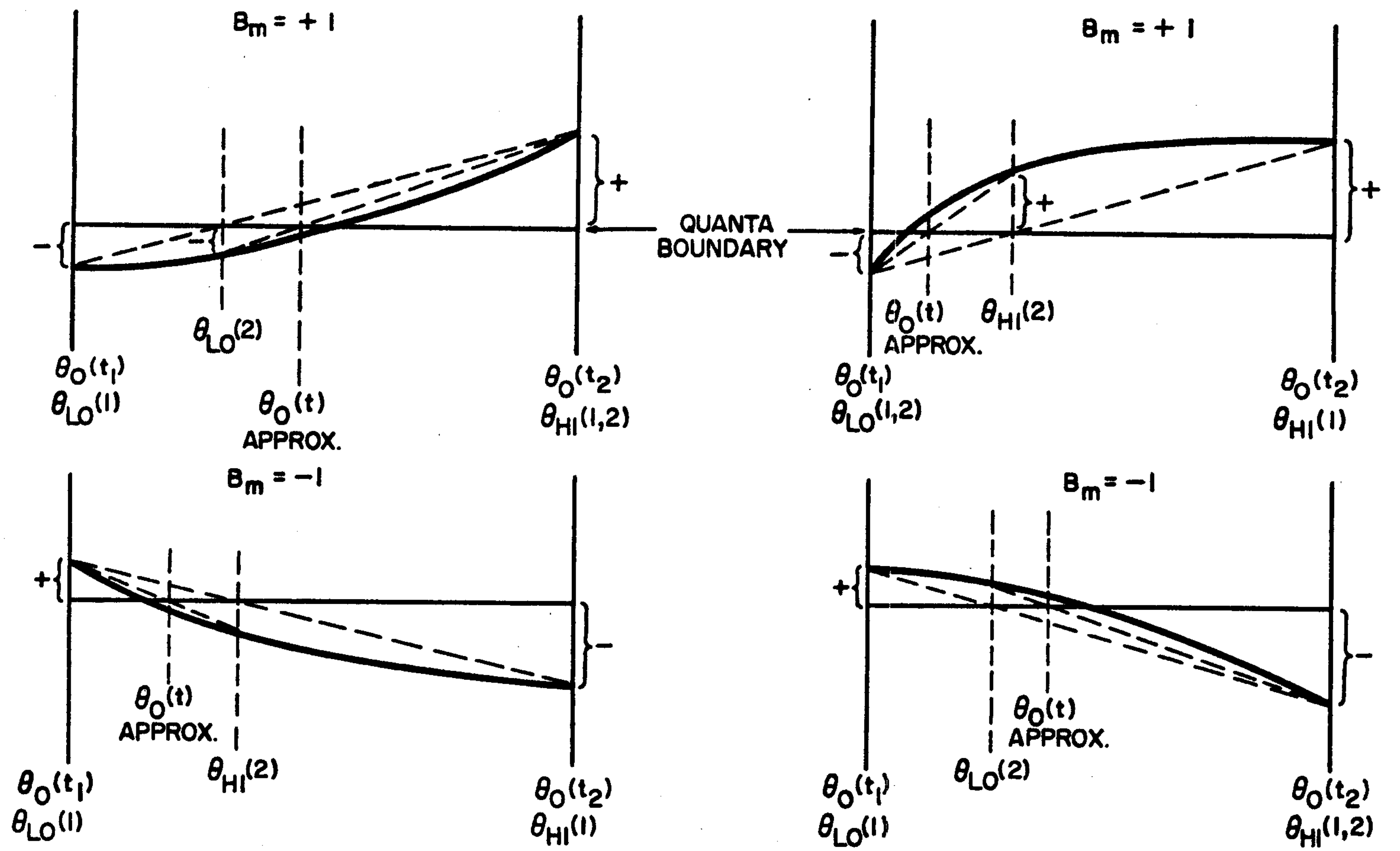


FIGURE 35 TIME DETERMINATION OF QUANTA BOUNDARY CROSSING

between θ_{HI} and θ_{LO} is continually reduced until $\theta_o(t)$ is within the prescribed error bounds.

Error Bound

The parameter, EPS, is used to define the allowable error range in the determination of the quanta boundary transition times. In the program utilized, it was required that the value of the effective boundary be within a fixed value EPS of the actual boundary. A slight difficulty, however, was encountered when the output approached a steady state value and the output began oscillating within the error bounds. In order to eliminate this difficulty and possibly improve the efficiency of the overall program, it is suggested that, rather than using a fixed value of EPS, it be allowed to vary inversely with the rate within specified limits; thus, for slower rates, a very small EPS would be used and for high rates, a large value of EPS should be utilized.

Selection of Time Increment

During the computation sequence, after each computation of $\theta_o(t)$, t will be increased by an amount Δt (unless a quanta boundary was crossed). The value of Δt was selected by taking one-half of the minimum anticipated time to travel from one quanta boundary to the next (maximum rate). It is suggested, however, that the computer time may be used more efficiently if a value of Δt is chosen which will vary within specified limits and which is inversely proportional to the rate. This is because it is necessary to compute only one to three values of $\theta_d(t)$ between each quanta point. It is estimated that the total computation time may be reduced by a factor of two or more using this technique.

General Comments

In conclusion, it may be noted that a subloop is provided to allow the output response to be determined for various values of τ and K_T without reading a new set of input data for each set of computations.

The computation time for a typical set of data (32 quanta step input) was approximately 30 minutes on the Burroughs 220 (using two values of Δt manually selected), but several sets of data were computed on the Univac 1107 approximately 5 minutes (using the minimum value of Δt).

BIBLIOGRAPHY

1. Kosem, M., "An Experimental Investigation of a Digital Position Servomechanism Incorporating Minimum Logic", M.S. Thesis, Case Institute of Technology, 1962.

2. Mergler, H. W., "Dynamic Digital Rate Measurement", Counter Points, Vol. 1, No. 4, pp. 1-2, Hawthorne, California: Erie-Pacific, Division of Erie Resistor Corporation.

3. Mergler, H. W., et. al., "Digital Control Systems Engineering", Case Institute of Technology Summer Program Notes, Cleveland, Ohio, 1962.

4. Ninke, W. H., "A Two-Dimensional Absolute Digital Data Path Control System", Doctor of Philosophy Thesis, Cleveland, Case Institute of Technology, 1964.

5. Taft, C. K., "Theory of Pulse-Data Systems Applied to an Input Self-Adaptive Pulse-Data System", Doctor of Philosophy Thesis, Cleveland, Case Institute of Technology.